Chapter 20

Foreign Currency Futures and Options

This chapter considers foreign currency futures and options and demonstrates how they can be used for hedging or speculative purposes. Because the profits and losses earned on futures and option contracts, as well as those earned on forward contracts, depend on how the spot exchange rate evolves over time, all these instruments are considered **derivative securities**. Derivative securities are securities whose values depend on the values of other, more basic underlying variables—in this case, the spot exchange rate.

As with other instruments in the foreign exchange market, much of the trade in futures contracts and options is conducted by banks. Commercial and investment banks deal aggressively in foreign currency options in order to meet the demands of their corporate and institutional customers, who use them to hedge their foreign exchange risks. In addition to banks, hedge funds and other investors trade foreign currency futures and options purely for speculative purposes—that is, strictly in order to earn a profit.

This chapter begins by introducing the foreign currency futures market and discussing how futures and forward contracts differ. It then discusses hedging with futures. Sections 20.3 and 20.4 present the basics of foreign currency options and their use in risk management. Section 20.5 examines some exotic options. As we first mentioned in Chapter 3, exotic derivatives caused a large number of international firms in emerging markets to suffer substantial losses during the 2007 to 2010 global financial crisis, and it is important to understand the risks involved.

20.1 The Basics of Futures Contracts

Futures Versus Forwards

Foreign currency futures contracts allow individuals and firms to buy and sell specific amounts of foreign currency at an agreed-upon price determined on a given future day. Although this sounds very similar to the forward contracts discussed in Chapter 3, there are a number of important differences between forward contracts and futures contracts.

Exchange Trading

The first major difference between foreign currency futures contracts and forward contracts is that futures contracts are traded on an exchange, whereas forward contracts are made by banks and their clients. Examples of futures exchanges include the CME Group with its CME Globex electronic trading platform, which trades G10 as well as a variety of emerging market currency futures; NYSE Euronext's LIFFE CONNECT, which trades dollar-euro futures and options; and the Tokyo Financial Exchange (TFX), which trades both yen-denominated futures as well as other base currency–denominated futures. With exchange trading, futures contracts are standardized by the exchange, whereas the terms of forward contracts are negotiable.

In addition, with exchange trading, orders for futures contracts must be placed during the exchange's trading hours. This used to be more of an issue when trading only occurred in the "pit" by floor traders. Now, electronic trading platforms match demand to supply over many more hours. For example, CME Globex operates 23 hours per day, only closing between 4 P.M. and 5 P.M. Central Time, during the work week. The exchange is closed from 4 P.M. Friday until 5 P.M. Sunday.

In contrast to forward contracts, where dealers quote bid and ask prices at which they are willing either to buy or sell a foreign currency, for each party that buys a futures contract, there is a party that sells the contract at the same price. The price of a futures contract with specific terms changes continuously, as orders are matched on the floor or by computer.

Standardized Amounts

Contracts on a futures exchange represent standardized amounts of currencies. Although the futures contracts cannot be tailored to a corporation's specific needs as can forward contracts, the standardized amounts are relatively small compared to a typical forward contract. If larger positions are desired, one merely purchases more contracts. The small contract sizes facilitate trade and enhance market liquidity. Some examples of the current contract sizes at the CME Group for currencies versus the dollar are JPY12,500,000, EUR125,000, CAD100,000, GBP62,500, CHF125,000, and AUD100,000. Other dollar-based contracts are also traded, ranging from the Swedish krona to the New Zealand dollar to a number of emerging market currencies, including the Mexican peso, the Brazilian real, and the Russian ruble. Cross-rate products, such as GBP/EUR or JPY/EUR, are also traded.

Fixed Maturities

In the forward market, a client can request any future maturity date, and active daily trading occurs in contracts with maturities of 30, 60, 90, 180, or 360 days. Contracts on futures exchanges have only a few maturity dates. For example, CME Group contracts mature on the third Wednesday of March, June, September, and December. These dates are fixed, and hence the time to maturity shrinks as trading moves from one day to the next, until trading begins in a new maturity. Typically, contracts are introduced 6 months before maturity. Consistent with the delivery procedures on spot foreign exchange contracts, trading in futures contracts stops at 9:16 A.M. Central Time 2 business days before the maturity day of the contract.

Credit Risk

As the box on the origins of the CME Group futures contracts indicates, banks willingly trade forward contracts with large corporations, hedge funds, and institutional investors, but they might not trade forward contracts with individual investors or small firms with bad credit risk.

One major reason why futures markets exist is the way credit risk is handled. In contrast to forward markets, in which the two counterparties must directly assess the credit risk of their counterparty, all contracts on a futures exchange are between a member of the exchange and the exchange itself. Retail clients buy futures contracts from futures brokerage firms, which in the United States must register with the **Commodity Futures Trading Commission (CFTC)** as a **futures commission merchant (FCM)**. Legally, FCMs serve as the principals for the trades of their retail customers. Consequently, FCMs must meet minimum capital requirements set

The Origins of Currency Futures

Although the Chicago Mercantile Exchange (CME), a precursor of CME Group, began trading agricultural futures contracts in 1898, the first foreign currency futures contract was not traded until 1972. It was done via the International Monetary Market (IMM), which was a subdivision of the CME. The IMM was the brainchild of Nobel Laureate Milton Friedman of the University of Chicago and Leo Melamed, the head of the CME, which was a world center for trading commodity futures. In the late 1960s, Friedman became convinced that the Bretton Woods system of fixed exchange rates was doomed. He predicted that the dollar would devalue relative to various European currencies, including the Deutsche mark, after the breakup.

Friedman wanted to profit from the situation, which he correctly foresaw, but he was frustrated by his attempts to purchase Deutsche mark–denominated forward contracts at a bank because he had no "legitimate" business purpose for doing so—aside from speculating, which banks frowned upon. Consequently, he approached Melamed about having the CME develop futures contracts for foreign currencies in which the average citizen could "vote with his dollars" on the government policies being discussed in Washington and other capitals around the world. Melamed liked the idea, and by 1972, foreign currency futures contracts were approved for trading by the Commodity Futures Trading Commission, and the IMM was born. Unfortunately for Friedman, the breakdown of Bretton Woods and the devaluation of the dollar occurred in August 1971, when President Richard M. Nixon withdrew the commitment of the United States to redeem dollars for gold—before Friedman could place his bet.

by the exchanges and fiduciary requirements set by the CFTC. In addition, if an FCM wants to trade with the CME Group, it must become a **clearing member**.

When a trade takes place on the exchange, the **clearinghouse** of the exchange, which is an agency or a separate corporation of the futures exchange, acts as a buyer to every clearing member seller and a seller to every clearing member buyer. The clearinghouse imposes margin requirements, which are also called performance bonds, and conducts the daily settlement process known as marking to market that mitigates credit concerns. These margin requirements are then passed on to the individual customers by the futures brokers.

Margins

When someone enters a forward contract, no money changes hands, and the only cash flow is at the maturity of the contract. Assessing credit risk is thus very important. When a futures contract is purchased or sold, the investor must deposit some assets into a **margin account** to fulfill the **initial margin** requirement and ensure that any future losses on the contract will be covered. These assets act as a **performance bond** because they may be confiscated if the investor loses money in the trade.[1] As futures prices change, one party to the contract experiences profits, and the other party experiences losses. The daily profits and losses are deposited to and subtracted from the margin accounts of the respective parties. This is the **marking to market** process that we will examine in detail shortly.

Clearing members of the CME clearinghouse accept margin payments in the form of cash, U.S. government obligations, securities listed on the NYSE or the American Stock Exchange (valued at 70% of their market prices), gold warehouse receipts (valued at 70% of the afternoon price of gold on the London Stock Exchange), or letters of credit of at least

[1]The CME Group uses the terms "margin" and "performance bond" interchangeably. In 1988, the CME Group developed the **Standard Portfolio Analysis of Risk (SPAN)** system, which calculates performance bond requirements for portfolios of positions using simulations of market prices. In discussing SPAN on its Web site, the CME Group states, "It is the official performance bond (margin) mechanism of 50 registered exchanges, clearing organizations, service bureaus and regulatory agencies throughout the world. SPAN software is utilized by a wide range of end-users, including futures commission merchants (FCMs), investment banks, hedge funds, research organizations, risk managers, brokerage firms and individual investors worldwide."

the amount required for the initial margin. It is important to realize that depositing assets in a margin account is not a payment for the futures contract. The investor still owns the assets that are in the margin account and can receive interest on monies deposited in the account.

In 2011, initial margins on the CME Group for members or hedgers were between $1,500 for the USD/GBP to $4,500 for the JPY/USD, and the **maintenance margins** were the same.[2] For speculators, the initial margins were higher. For example, the JPY/USD contract required $6,075 as initial margin and $4,500 as maintenance margin. The maintenance margin is the minimum amount that must be kept in the account to guard against severe fluctuations in the futures prices and the losses that would be incurred by one of the parties. When the value of the margin account reaches the maintenance margin, there is a **margin call**, at which point the account must be brought back up to its initial value. Because margins are intended to control risk, their magnitude depends on the size of the contract and the volatility of the currencies.

Of course, the initial margin payments must eventually reach the clearinghouse. This happens through a pyramid structure. The clearinghouse, which sits at the top of the pyramid, collects the margins from clearing member FCMs, which collect the margins from non-clearing member FCMs, which collect them from their customers and execute their trades through FCM clearing members.

Marking to Market

The system of margin accounts coupled with a process of daily marking to market ensures that the users of these contracts present little credit risk to the FCMs and thus to the clearinghouse of the exchange. To better understand the process, let's examine marking to market, using a euro futures contract. Let's assume that each contract represents €125,000.

Suppose it is September, and you buy a December euro futures contract. Buying the contract means that you "go long in December euro," and you will profit if the euro appreciates relative to the dollar. Conversely, you will take losses if the euro depreciates. You place your order to buy with your broker, and the order is executed on the futures exchange at a price at which another trader is willing to sell the identical contract. This trader could be selling for his own account, or he could be executing an order on behalf of someone who has placed an order to "short" the December euro contract.

Consider how the contractual profits and losses evolve over time and how this affects your margin account. Exhibit 20.1 provides a 7-day example. Suppose that your trade was

Exhibit 20.1 An Example of Marking to Market in the Futures Market

Day	Futures Price ($/€)	Change in Futures Price ($/€)	Gain or Loss	Cumulative Gain or Loss	Margin Account
t	1.3321	0	0		$2,000.00
t+1	1.3315	−$0.0006	−$75.00	−$75.00	$1,925.00
t+2	1.3304	−$0.0011	−$137.50	−$212.50	$1,787.50
t+3	1.3288	−$0.0016	−$200.00	−$412.50	$1,587.50
t+4	1.3264	−$0.0024	−$300.00	−$712.50	$2,000.00
t+5	1.3296	+$0.0032	+$400.00	−$312.50	$2,400.00
t+6	1.3301	+$0.0005	+$62.50	−$250.00	$2,462.50

Notes: The futures price column lists the daily settle prices in the futures market. The contract size for the euro contract is assumed to be €125,000. The initial margin is $2,000, and the maintenance margin is $1,500. The gain or loss is the change in the futures price ($/€) multiplied by the size of the contract. The cumulative gain or loss is the sum of the daily gain or loss.

[2]Current margin requirements may be found at www.cmegroup.com.

filled on September 16, at the end of trading, and the **settle price**, or final futures trading price, for that day for the December contract was \$1.3321/€. When you purchase the December euro contract, you must place the initial margin, which is assumed to be \$2,000, into your margin account. The individual who sold the euro contract to you must also place \$2,000 into his or her margin account. We assume that the maintenance margin is \$1,500. In other words, if the value of your margin account drops by more than \$500 because of losses on your futures position, you will be required to bring the account's balance back up to the initial \$2,000.

Suppose that on September 17, the dollar price of the December euro futures contract falls by \$0.0006/€, to \$1.3315/€. This is the new daily settle price of the contract, and it affects the balance in your margin account. Because you are long in the euro contract, and the futures price of the euro fell, money is taken out of your margin account. Conversely, the person who sold the December futures contract—that is, the one who shorted the euro—gains money. The amount taken from your margin account to be placed in the margin account of the individual who sold the euro short is the change in the settle price times the contract size:

$$(\$0.0006/€) \times €125{,}000 = \$75$$

This process continues every day, until the maturity date of the contract. Exhibit 20.1 indicates that if the euro futures price falls to \$1.3264/€ by day t+4, you will have a cumulative loss of \$712.50. Because this cumulative loss makes the value of your margin account less than the maintenance margin of \$1,500, you will receive a margin call from your broker, notifying you that you have to increase your margin account back to the initial margin of \$2,000. Exhibit 20.1 also indicates that funds will be credited to your margin account if the December futures price increases as it does on days t+5 and t+6. These funds could either be left in your account, as in Exhibit 20.1, or they could be withdrawn to leave the margin account at the value of the initial margin (\$2,000).

On the last trading day of the futures contract, 2 business days remain before delivery. Trading the futures contract stops at 9:16 A.M. Central Time. Arbitrage guarantees that the futures price at the maturity of the contract will be equal to the spot exchange rate on that day because both the futures price and the spot price are ways of purchasing euros with dollars for delivery in 2 business days. Hence, if on the Monday before the third Wednesday of December, the spot price is \$1.3421/€, the futures price will have risen by

$$1.3421/€ - \$1.3321/€ = \$0.0100/€$$

You will have had an inflow of profit equal to

$$(\$0.0100/€) \times €125{,}000 = \$1{,}250$$

Of course, because you received the \$1,250 in increments, it will actually be worth something slightly more than this amount because you will have received interest on your profits.

The marking-to-market process means that entering a futures contract can be thought of as a sequence of bets on the direction of the change in the price of the contract rather than a direct future purchase of foreign currency. This is an accurate description because all gains and losses are settled every day. Each day, you face the decision of sticking with your long or short position, which is called your **open interest**, or ending the bet by taking the reverse position. If you are long one contract and you sell one contract for the same maturity, the clearinghouse simply nets your position to zero. This is the way most futures contracts are closed out.

The Pricing of Futures Contracts

Because forward and futures contracts both allow you to buy or sell foreign currency at a particular future time at an exchange rate known today, you might think that the two prices should be the same. However, because forward contracts entail no cash flows until maturity, whereas futures contracts are marked to market, the two prices can, in theory, be slightly different.

Comparing Payoffs

Let's illustrate the payoff patterns for forward and futures contracts in symbols. Let $F(t)$ be the forward price of the foreign currency at time t. Then, the payoff per unit of foreign currency at maturity, time T, depends on the future spot rate, $S(T)$. If you purchase the foreign currency forward, the payoff equals

$$S(T) - F(t)$$

You win if $S(T) > F(t)$, and you lose if $S(T) < F(t)$.

Suppose you buy a foreign currency futures contract at time t at the futures price, $f(t)$, and you hold the contract until maturity, the same time T as the maturity of the forward contract. Because the payoff dribbles in over time due to marking to market, the per-unit payoff is

$$\begin{array}{rl}
\text{Day } t+1: & f(t+1) - f(t) \\
\text{Day } t+2: & f(t+2) - f(t+1) \\
\text{Day } t+3: & f(t+3) - f(t+2) \\
 & \vdots \\
\text{Day } T: & f(T) - f(T-1)
\end{array}$$

If we ignore the time value of money and add up all the cash flows, the aggregate payoff is

$$[f(t+1) - f(t)] + [f(t+2) - f(t+1)] + [f(t+3) - f(t+2)] + \ldots$$
$$+ [f(T) - f(T-1)] = f(T) - f(t)$$

because the intermediate futures prices cancel out. Because arbitrage drives the futures price at maturity, $f(T)$, to equality with the spot rate on that day, $S(T)$, the payoff on the futures contract is essentially the same as the payoff on the forward contract.

Why Futures Can Differ from Forwards

The payoffs of futures contracts and forward contracts are only "essentially the same" because a slight difference in payoffs arises when we do not ignore the interest that is earned on future profits or that must be paid on future losses. Technically, if the path of short-term interest rates could be foreseen—that is, if there were no random elements in the change in future short-term interest rates—there would be an arbitrage possibility if the forward exchange rate were different from the futures price because you would know how you could invest the profits or borrow to finance your losses. However, future interest rates are not known with certainty, so forward prices and futures prices can be different, in theory. In practice, though, the price differentials are minimal, and they appear to be within the transaction costs of the forward market. Therefore, we argue that futures prices are "essentially the same" as forward prices, and we don't explore further how futures contracts are valued.

Futures Quotes

Now that you understand how futures markets work, let's examine Exhibit 20.2, which shows data on futures prices from the CME Group. The information reports trading from April 21, 2011, and the first trade, which is the **open price** on that day, for a June euro contract was \$1.4505/€.

During the day, trades occurred at prices as high as \$1.4631/€ and as low as \$1.4487/€. The settle price was \$1.4555/€, and this price represents the value weighted price for all trades conducted in the last 30 seconds before 2:00 P.M. Central Time.

Exhibit 20.2 Futures Quotes from April 21, 2011

Contract Size	JPY12,500,000			CAD100,000			GBP62,500			EUR125,000		
Exchange Rate	**USD per 100 JPY**			**USD per CAD**			**USD per GBP**			**USD per EUR**		
Maturity	**JUN**	**SEP**	**DEC**	**JUN**	**SEP**	**DEC**	**JUN**	**SEP**	**DEC**	**JUN**	**SEP**	**DEC**
Open Price	1.2111	1.2136	1.2193	1.0486	1.0475	1.0478	1.6387	1.6460	—	1.4505	1.4454	1.4444
High Price	1.2256	1.2261	1.2257	1.0563	1.0534	1.0502	1.6590	1.6562	—	1.4631	1.4585	1.4510
Low Price	1.2111	1.2125	1.2138	1.0472	1.0447	1.0416	1.6376	1.6356	—	1.4487	1.4447	1.4444
Settle Price	1.2232	1.2241	1.2254	1.0491	1.0463	1.0431	1.6538	1.6514	—	1.4555	1.4512	1.4468
Change in Price	0.0093	0.0093	0.0093	0.0027	0.0026	0.0024	0.0141	0.0141	—	0.0058	0.0058	0.0060
Open Interest	125,635	1,312	119	139,545	2,781	2,481	112,115	430	—	246,358	3,137	86

Contract Size	CHF125,00			AUD100,00			MXN500,000			EUR100,000		
Exchange Rate	**USD per CHF**			**USD per AUD**			**USD per 10 MXN**			**JPY per EUR**		
Maturity	**JUN**	**SEP**	**DEC**	**JUN**	**SEP**	**DEC**	**JUN**	**SEP**	**DEC**	**JUN**	**SEP**	**DEC**
Open Price	1.1256	1.1275	—	1.0636	1.0535	—	0.8575	—	—	119.39	—	—
High Price	1.1392	1.1393	—	1.0710	1.0578	—	0.8605	—	—	119.39	—	—
Low Price	1.1249	1.1257	—	1.0613	1.0485	—	0.8545	—	—	118.83	—	—
Settle Price	1.1324	1.1329	—	1.0681	1.0550	—	0.8580	—	—	119.02	—	—
Change in Price	0.0071	0.0072	—	0.0077	0.0076	—	0.0008	—	—	-0.38	—	—
Open Interest	71,895	123	—	146,579		—	172,588	—	—	7,242	—	—

Note: All contracts except the JPY/EUR are traded on the CME Group. The JPY/EUR contract is trade on ICE Futures U.S. Data sources are Thomson Reuters and the Wall St. Journal Market Data Group.

Futures Contracts for Emerging Markets

In addition to trading futures contracts on the major currencies of the world, the CME Group now trades quite a few contracts on emerging-market currencies. The first of these to be established was for the Mexican peso, which began trading in April 1995.

At that time, trading futures contracts for the Mexican peso was quite a courageous move. Mexico had just witnessed a severe currency crisis, and Larry Summers, the U.S. Secretary of the Treasury, argued that introducing Mexican peso futures would be a bad idea because it would be easier for currency speculators to bet against the Mexican peso. Moreover, when plans for the contract were unfolding, it became clear that the usual delivery procedures of the CME were incompatible with the capital controls in place in Mexico.

In addition, the CME wanted to involve the Mexican government in the process of establishing the contract. Fortunately, the Mexican minister of finance at the time, Guillermo Ortiz, a Stanford-trained economist, thought that the introduction of a CME Mexican peso futures contract fit in well with his plans to restore confidence in the Mexican government and economy and to move toward more market-oriented policies. Ortiz argued that an effective futures contract would be hugely beneficial to international trade between the United States and Mexico because it would facilitate hedging, and he did not feel it would generate excessive exchange rate volatility. In fact, Ortiz decided to lift the Mexican capital controls, making it possible for the CME to employ its usual delivery procedures for the Mexican peso contract when it launched in April 1995.

This turned out to be a good decision in facilitating the success of the contract because actual delivery of currency was used more often than is the case with major currencies. That is, many of the users of the contracts turned out to be exporters and importers who desired the actual delivery of the currencies.

For the CME, this was the beginning of an Emerging Markets division that now has contracts listed not only on the Mexican peso but also on the Brazilian real, the Russian ruble, the Czech koruna, the Hungarian forint, the Polish zloty, the Chinese renminbi, the Korean won, the Israeli shekel, the Turkish lira, and the South African rand. The CME Group now also trades euro-denominated contracts on the koruna, the forint, the zloty, the renminbi, and the lira.

The row labeled “Change in Price” in Exhibit 20.2 indicates that the new settle price is \$0.0058/€ higher that the previous day’s settle price.

The final row represents the open interest that is outstanding, which is 246,358. The open interest is the number of pairs of contracts bought and sold that have not yet been closed out. Notice that the largest open interest is in the contract closest to maturity. This is typically true until the contract enters the maturity month, in which case activity switches to the next-closest contract.

20.2 Hedging Transaction Risk with Futures

This section examines how futures contracts can be used to hedge exposures to transaction exchange risk. It does so in the context of an extended example.

Hedging at Nancy Foods

Suppose it is the middle of February, and Nancy Foods, an American firm, has just contracted to sell frozen quiches to Kühlerkuchen, a German firm. Nancy Foods will receive €250,000 in the middle of March and is considering hedging the exposure with futures contracts.

The Hedging Decision

First, because the contract size on the CME Group is €125,000, Nancy Foods uses two contracts. Second, Nancy Foods has to determine whether it wants to buy or sell the futures contracts. Because it has a €250,000 account receivable, which is a euro asset, Nancy Foods will lose money if the euro weakens relative to the dollar. The company will gain if the euro strengthens relative to the dollar. Consequently, to hedge its exposure, Nancy Foods must enter into futures contracts that provide profits when the euro weakens and losses when the euro strengthens. That is, Nancy Foods hedges by acquiring a euro liability whose value is equivalent to the value of the underlying receivable.

If Nancy Foods sells two euro futures contracts, it profits when the euro weakens because the dollar value of €250,000 in the futures market is going down. The company loses on the futures contract if the euro strengthens because the dollar value of €250,000 in the futures market is going up. Notice that if the maturity date of the receivable is the third Wednesday in March, the maturity of the euro asset from the underlying receivable and the euro liability represented by Nancy Foods's sale of the futures contracts are matched exactly. Hence, the company is effectively hedged.

A Numeric Example

To be concrete, let's assume that the following exchange rates are observed:

	Spot Rate	Futures Rate (March contract)
February	\$1.24/€	\$1.23/€
March	\$1.35/€	\$1.35/€

The March futures rate coincides with the spot rate because both are for the third Wednesday in March. Because Nancy Foods is exposed to euro depreciation, the company goes short two futures contracts, at the futures rate of \$1.23/€. What are the final cash flows?

First, when Nancy Foods sells the euro receivables in the spot market in March, the cash flow is

$$€250{,}000 \times \$1.35/€ = \$337{,}500$$

Second, the futures contract will have lost money because Nancy Foods established a short position in the futures market, and the euro appreciated versus the dollar. The cash flow on the futures contract is the change in the futures price multiplied by the contractual amount:

$$[(\$1.23/€) - (\$1.35/€)] \times €250{,}000 = -\$30{,}000$$

Combining the cash flow from the euro receivables with the loss on the futures contracts yields a total cash flow of

$$\$337{,}500 - \$30{,}000 = \$307{,}500$$

The effective exchange rate at which Nancy Foods sells the euro receivables is

$$\$307{,}500/€250{,}000 = \$1.23/€$$

Thus, by transacting in the futures market, Nancy Foods effectively locks in the original futures price.

Potential Problems with a Futures Hedge

Hedging transactions exposures with futures has two obvious problems. First, futures contracts are sold only in standardized sizes (€125,000 in our example). Hence, if you need to hedge an amount that is not a multiple of the standard size, some of your risk cannot be covered. A second problem is caused by the relatively low number of delivery dates. If the maturity of your foreign currency asset or liability does not match a settlement date in the futures market, the relationship between the spot exchange rate at the time the transaction takes place and the futures price of the foreign exchange is somewhat uncertain.

Basis Risk

To provide a perfect hedge, the price of the futures contract should move one-for-one with the spot exchange rate. Then, being long in the foreign currency from an underlying transaction can be hedged by going short in the corresponding futures contract. If this is not the case, the hedge is said to suffer **basis risk**. The basis is the difference between the spot price at time t, $S(t)$, and the futures price at time t, $f(t,T)$, for maturity date at time T:

$$\text{Basis} = \text{Spot price} - \text{Futures price} = S(t) - f(t, T)$$

Mostly, we refer to a single maturity, so we will omit the T indicator.

To see how the basis affects the quality of a hedge, let's ignore the time value of money because the maturity is short, and let's consider how the value of the receivable and the hedge move over time in Exhibit 20.3. Initially, the value of the receivable per unit of foreign currency is worth $S(t)$, the value of the spot exchange rate. The problem is that you can only sell the receivable at time T at the as-of-yet unknown exchange rate $S(T)$. The uncertain change in value $S(T) - S(t)$ represents your transaction exchange risk. Column 2 in Exhibit 20.3 shows how the value of the receivable moves with the spot rate. When you hedge a foreign currency asset using the futures market, you sell the foreign currency futures. Initially, the futures contract has no value, but on day 2, the cash flows start coming in (or leaving) your margin account. We record the cumulative cash flows in the third column. The fourth column reflects the value of the hedged position: the receivable plus the cash flows earned or lost in the futures market. It is easy to see that the hedged position equals the futures rate at which you entered the contract plus the basis. Consequently, to make sure you really lock in the future rate, the basis at maturity must be zero.

Suppose we hold the contract until maturity. In that case, the futures rate converges to the spot rate; that is, the basis is zero at maturity. Then, Exhibit 20.3 shows that the hedged position is worth $f(t)$; you effectively sell the receivable at the futures rate. If the maturities of the receivable and the futures contract do not coincide, the basis will not equal zero when the futures contract is closed, and there will be basis risk. Note that the value of the hedged position has changed as follows between time t and time T:

$$f(t) + [S(T) - f(T)] - S(t) = [S(T) - f(T)] - [S(t) - f(t)]$$

Hence, the change in value in the hedged position equals the change in basis between time t and T. If the basis is zero at maturity, this change in value is perfectly known at time t. Although basis risk is typically much smaller than the risk associated with an uncovered

Exhibit 20.3 Hedging a Receivable with Futures

Time	Value of Receivable	Cumulative Value of Futures Hedge (short position)	Value of Hedged Position
t	$S(t)$	0	$S(t)$
$t+1$	$S(t+1)$	$f(t) - f(t+1)$	$f(t) + [S(t+1) - f(t+1)]$
$t+2$	$S(t+2)$	$[(f(t+1) - f(t+2)] + [f(t) - f(t+1)]$ $= f(t) - f(t+2)$	$f(t) + [S(t+2) - f(t+2)]$
$\vdots$		$\vdots$	$\vdots$
T	$S(T)$	$f(t) - f(T)$	$f(t) + [S(T) - f(T)]$

Note: The hedged position reflects the sum of the previous two columns.

position, a substantial amount of risk may nevertheless remain. Risk managers often use quantitative techniques to figure out the best way to mitigate basis risk, but these techniques are beyond the scope of this book.

Example 20.1 A Euro Receivable and Basis Risk

Let's return to the situation in which Nancy Foods is contracting to sell quiches in Germany, thereby generating a €250,000 receivable. This time, assume that the contract is made in January, and payment is scheduled for early March. Now, the delivery date for the quiches does not coincide with the maturity date of the futures contract, and Nancy Foods consequently faces basis risk. We assume that the following exchange rates are observed:

	Spot Rate	Futures Rate (March contract)
January	\$1.21/€	\$1.22/€
March	\$1.33/€	\$1.325/€

To protect itself from euro depreciation, Nancy Foods sells two futures contracts at the futures rate of \$1.22/€. What are the final cash flows now?

First, Nancy Foods sells the euro receivables in the spot market, receiving

$$€250{,}000 \times \$1.33/€ = \$332{,}500$$

Second, the futures contract lost money because the euro appreciated, and Nancy Foods established a short position in the futures market. The total cash flow would be

$$[(\$1.22/€) - (\$1.325/€)] \times €250{,}000 = -\$26{,}250$$

So, ultimately, the euro receivables plus the loss on the futures contract yields

$$\$332{,}500 - \$26{,}250 = \$306{,}250$$

The effective exchange rate at which Nancy Foods sold the euro receivables is

$$\$306{,}250/€250{,}000 = \$1.225/€$$

This does not equal the futures rate of \$1.22/€ because of basis risk. The difference of \$0.005/€ with the futures rate exactly reflects the basis (Spot rate − Futures rate = \$1.33/€ − \$1.325/€) at the time that the futures contract was closed out and the receivable sold for dollars in the spot market. In this case, basis risk had a positive effect on Nancy Foods's cash flow. That is, we have, as in Exhibit 20.3,

$$\text{Effective rate} = \text{Futures rate} + \text{Basis}$$
$$\$1.225/€ = \$1.22/€ + (\$1.33/€ - \$1.325/€)$$

After the fact, we see that Nancy Foods would have been better off not hedging at all because the euro actually appreciated, and the company had a euro receivable. If Nancy Foods wanted to hedge completely, though, the futures market works pretty well—even in the presence of basis risk.

In Section 20.3, we will look at how options allow companies to hedge while retaining some benefit from advantageous exchange rate movements. But first, we need to see how the Handel brothers are doing.

POINT–COUNTERPOINT

On Good Beer and Korunas

The Handel family reunion on Uncle Fred's estate in Chappaqua, New York, brought Ante, Freedy, and Suttle together again with their flamboyant uncle who's in the export–import business. Uncle Fred was keen to get his nephews' insights on the international financial issues he faced. After dinner, he insisted that they all meet at the bar in his den because he had something to show them. Uncle Fred poured a particularly clear lager from a funky-looking bottle and roared, "Here my friends, drink this!"

"What beer is this?" Ante inquired, "This tastes wonderful!"

"Well, my friends, this is Pilsner Kozquell, an authentic Czech lager," their uncle explained. "It is brewed under strict purity laws—only hops, yeast, malt, and water can be used. The result is very different from that chemically, carbon dioxide–infused, scrub water they make as beer in America! And guess what? This wonderful beer may soon be available in America at reasonable prices, as I am hoping to start importing the stuff! I have bid for the import license with the Czech brewery, and if everything goes well, the first shipment should arrive in 6 months."

"That's wonderful news, Uncle!" exclaimed Freedy.

"Well, there are problems," sighed their uncle. "I'm not sure I'm going to win the bid, and the brewery will take a month to decide. Moreover, they insist on being paid in Czech korunas. I've got a potentially huge koruna liability 6 months from now, and I am worried about the currency risk. I was kind of hoping you guys could help me out. What can I do to hedge this risk? At current exchange rates, my margins are not that great, and I cannot afford to pay many more dollars for the beer. On top of all that, the dollar has been weakening, and my bank is not willing to do a koruna forward contract with me. They say I've maxed out my credit limit."

"Ha," said Ante, "I would not hedge! The Czech Republic is now a member of the European Union, and it may soon adopt the euro as its currency. Because it is an emerging market, it likely still has tons of inflationary pressure, and I suspect its currency will depreciate tremendously in the run-up to adopting the euro. If that happens, your liability will be melting away in dollar terms if the dollar stays even with the euro."

"No way, Uncle! Don't take that risk!" Freedy interjected. "If the koruna moves with the euro versus the dollar, the opposite may happen. Also, the koruna might appreciate against both currencies, as people hoard it in anticipation of joining the euro monetary system. With the weak dollar, your koruna exposure is very risky now! I would use the futures market to hedge. The CME Group has futures contracts on the koruna, so you can go long in koruna futures. If the koruna appreciates, the payment for the shipment is going to cost you more dollars, but the futures position will gain money, too, offsetting the loss on the payable."

"That sounds interesting," Uncle Fred mused. "But, Suttle, tell me what you think."

Suttle reluctantly put down his glass of Pilsner Kozquell and said, "I think there is indeed a chance the koruna will depreciate as Ante claims, but I've heard that the Czech economy is doing very well, and the currency has been stable. In fact, the Czech central bank has competently adopted a modern monetary policy, and the inflation rate there has recently run at a lower rate than in the European Union. Hence, the risk of koruna appreciation versus the euro is real. The risk of euro appreciation versus the dollar is also very real. With such risks and low profit margins, some form of hedge is probably a good idea. However, it depends on your point of view. How sure are you that you will win the contract? If you hedge with a long koruna futures position and don't get the contract, you'll take losses if the koruna weakens. I think you need to look into options. Because you need to buy koruna, why not buy a koruna call option? You pay a bit of a premium, but you are hedged, and you still profit from a lower dollar payment in case the koruna depreciates. If you don't get the contract, the most you can lose is the option premium, and these options also trade at the CME."

Ante gasped: "Options? Gee, I've got to study this for our international finance exam. They're so complicated!" Uncle Fred just smiled and poured another Pilsner Kozquell lager. He knew what to do.

20.3 Basics of Foreign Currency Option Contracts

A foreign currency option contract gives the buyer of an option the right, but not the obligation, to trade a specific amount of foreign currency for domestic currency at a specific exchange rate. Foreign currency options are traded primarily over the counter (OTC) by money center banks, but they are also traded on organized exchanges. Two of the largest exchanges are the NASDAQ OMX PHLX, which was formed in 2008 when the Philadelphia Stock Exchange (PHLX) was purchased by NASDAQ OMX, and the International Securities Exchange, which is a subsidiary of Eurex.

Basic Option Terminology

The two fundamental types of options are calls and puts. A **foreign currency call option** gives the buyer of the option the right, but not the obligation, to buy a specific amount of foreign currency with domestic currency at an exchange rate stated in the contract. A **foreign currency put option** gives the buyer of the option the right, but not the obligation, to sell a specific amount of foreign currency for domestic currency at an exchange rate stated in the contract. Because the buyer of the option purchases the right to transact from the seller, the buyer must pay the seller the value of the option, which is the option's price. Market participants also refer to the option price as an **option premium**. The seller of the option is also referred to as the writer of the option.

European Versus American Options

Foreign currency option contracts have an expiration or maturity date. If the buyer of an option decides to engage in the transaction at the time specified in the option contract, she is said to have "exercised" her option. If the buyer has not exercised her option by the expiration date, the option becomes worthless. An option that can be exercised only at maturity is called a **European option**. An option that can be exercised at the discretion of the buyer at any time between the purchase date and the maturity date is called an **American option**. If an American option is exercised prior to maturity, the person is said to have engaged in **early exercise**.[3]

Strike Prices and Intrinsic Value

The exchange rate in an option contract is called the option's **strike price**, or **exercise price**. Investors commonly compare a contract's strike price with the current spot exchange rate. If some revenue could be earned by exercising the option immediately, even though the option holder cannot or might not want to exercise it, the option is said to be

[3]Note that the terminology describing when options can be exercised—that is, European vs. American—has nothing to do with where the options are traded or how the exchange rates are quoted. The terminology only describes the inability (European) or ability (American) of the buyer to exercise the option prior to maturity.

"in the money." If no revenue could be earned by exercising the option immediately, the option is said to be "out of the money." An "at-the-money" option has a strike price equal to the current spot rate. Traders also speak of options that are "at-the-money-forward" if the strike price is equal to the forward rate for that maturity. Option transactions can also be terminated by closing out the position in the OTC market, or by reversing the original transaction, as in the futures markets. That is, the buyer of the option can simply sell the contract on the exchange.

The immediate revenue from exercising an option is called the option's **intrinsic value**. Let K be the strike price, and let S be the current spot rate, both in domestic currency per unit of foreign currency. Then, the intrinsic value per unit of a foreign currency option can be represented as

$$\text{Call option intrinsic value: } \max[S - K, 0]$$
$$\text{Put option intrinsic value: } \max[K - S, 0]$$

Here, max denotes the operation that takes the maximum of the two numbers between the square brackets. For example, when the spot rate is smaller than the stock price ($S < K$), the call option is not worth exercising immediately, so its intrinsic value is 0, but the put option's intrinsic value is $K - S$. Now that we have examined the terminology of options, let's look at some concrete examples.

Example 20.2 A Euro Call Option Against Dollars

A euro call option against dollars gives the buyer the right, but not the obligation, to purchase a certain amount of euros, such as €1 million, with dollars at a particular exchange rate, such as \$1.20/€. If the spot exchange rate of dollars per euro in the future is greater than the exercise price of \$1.20/€, the buyer of the option will exercise the right to purchase euros at the lower contractual price. When exercising the option, the buyer pays the seller of the option

$$(\$1.20/€) \times €1{,}000{,}000 = \$1{,}200{,}000$$

and the seller delivers the €1,000,000. The buyer of the option can then sell the euros in the spot market for dollars at whatever spot rate, $S(\$/€)$, prevails at that time, generating dollar revenue of

$$S(\$/€) \times €1{,}000{,}000$$

Hence, the net dollar revenue generated for the buyer of the option is equal to the difference between the current spot price and the exercise price multiplied by the contractual amount. If the spot rate is \$1.25/€, the net dollar revenue from exercising the euro call option on €1,000,000 is

$$[(\$1.25/€) - (\$1.20/€)] \times €1{,}000{,}000 = \$50{,}000$$

Note that this is the intrinsic value of the option at the time of exercise, $\max[S - K, 0]$, multiplied by the contract size. Remember that the \$50,000 is purely the revenue from exercising the option. It is not the profit to the purchaser of the option because it does not subtract the cost of the option position.

Notice also that the right to buy €1,000,000 with dollars at the exchange rate of \$1.20/€ is equivalent to the right to sell \$1,200,000 for €1,000,000. This option is described as a dollar put option against the euro with contractual amount of \$1,200,000 and a strike price of

$$€1{,}000{,}000/\$1{,}200{,}000 = 1/(\$1.20/€) = €0.8333/\$$$

These options are the same; they are just described differently.

Also, note that the buyer of the option could accept a payment of \$50,000 from the seller of the option to close out the position rather than take delivery of the €1,000,000 and resell the euros in the spot market. Many option contracts are closed in this way, and this is how options on the NASDAQ OMX PHLX are settled.

Example 20.3 A Yen Put Option Against the Pound

A Japanese yen put against the British pound in a European contract gives the buyer of the option the right, but not the obligation, to sell a certain amount of yen, say ¥100,000,000, for British pounds to the seller of the option at the maturity of the contract. The sale takes place at the strike price of pounds per 100 yen, say £0.6494/¥100. If the spot exchange rate of pounds per 100 yen at the exercise date in the future is less than the strike price, the buyer of the option will exercise the right to sell the ¥100,000,000 for pounds at the higher contractual price. When exercising the option, the buyer delivers ¥100,000,000 to the seller of the option, who must pay

$$(£0.6494/¥100) \times ¥100{,}000{,}000 = £649{,}400$$

Suppose that the spot exchange rate at maturity is £0.6000/¥100 yen, which is less than the strike price. Then, the buyer of the option can purchase ¥100,000,000 in the spot foreign exchange market for £600,000 and sell the yen to the person who wrote the put contract. By exercising the option, the buyer of the yen put generates pound revenue equal to the difference between the exercise price of £0.6494/¥100 and the current spot price of £0.6000/¥100 multiplied by ¥100,000,000:

$$[(£0.6494/¥100) - (£0.6000/¥100)] \times ¥100{,}000{,}000 = £49{,}400$$

This corresponds to the intrinsic value of the contract at maturity multiplied by the contract size—that is, Revenue $= \max[K - S, 0] \times$ Contract size. Once again, this is purely the revenue from the option contract; it is not the profit to the purchaser of the option because it does not subtract the original cost of the put option.

Notice, also, that the right to sell ¥100,000,000 for British pounds at the exchange rate of £0.6494/¥100 is equivalent to the right to buy £649,400 with yen at the exchange rate of

$$¥100{,}000{,}000/£649{,}400 = 1/(£0.6494/¥100) = ¥153.99/£$$

This latter option is a British pound call option against the Japanese yen.

Options Trading

Most options are traded by banks, either in the interbank market or as OTC transactions with the bank's clients. That is, transactions are done in a dealer network and are not listed on any centralized exchange. Typical OTC options use the European exercise convention. In the OTC market, though, a reasonable request by a corporate customer for any type of option with a particular strike price, maturity date, or other characteristic will be met with a price quoted by a bank. OTC options are also typically written for much larger amounts than exchange-traded options, and a much broader range of currencies is covered.

The cash flows generated by exercise of an OTC option are handled either by exchange of the relevant currency amounts 2 business days after the notification of exercise or, often, by cash settlement. In the latter case, the writer of the option compensates the buyer of the option for the revenue that the option generates when the option ends up in the money.

As with forward contracts, there is a considerable amount of counterparty risk that concerns both bank traders and corporate treasurers. Banks manage their counterparty risks by establishing maximum exposure limits to particular clients, and corporate treasurers must be aware of the risks of dealing with particular banks.

Currency Options on the NASDAQ OMX PHLX

The NASDAQ OMX PHLX trades options on spot currencies versus the U.S. dollar. The contracts specify different amounts of the underlying foreign currency: 10,000 units of foreign currency for the Australian dollar, the British pound, the Canadian dollar, the euro, the Swiss franc, and the New Zealand dollar; 100,000 units of foreign currency for the Mexican peso, the Norwegian krone, the South African rand, and the Swedish krona; and 1,000,000 Japanese yen. The expiration months are March, June, September, and December plus the 2 nearest future months. The last trading day is the third Friday of the expiring month. The exercise style is European. The settlement of all the contracts is in dollars. The option prices are quoted in U.S. cents per foreign currency unit for the currencies in which the contractual amount is 10,000 units; in 0.1 U.S. cents per unit for the currencies in which the contractual amount is 100,000 units; and in 0.01 U.S. cents for the yen. Thus, a one-point move in the option price corresponds to a gain or loss of $100 on each of the contracts. The Options Clearing Corporation serves as the official clearinghouse for options trades on the NASDAQ OMX PHLX. Let's consider an example from this market.

Example 20.4 A Euro Call Option Against Dollars

On October 1, 2010, the euro was trading at $1.3780/€. A euro call option with a strike price of "135" and a December 2010 maturity was quoted at 4.75 cents per euro. Because the strike price is expressed in cents per euro, we can convert it to dollars per euro, or $1.35/€, and a similar transformation of the option price gives $0.0475/€. For a contract size of €10,000, this option would have cost

$$(\$0.0475/€) \times €10{,}000 = \$475$$

While this option cannot be used to buy the euro, notice that the the cost of purchasing €10,000 at the stike price of $1.35/€ would have been

$$(\$1.35/€) \times €10{,}000 = \$13{,}500$$

Therefore, the option premium (the cost of the option) represents less than 4% of the value of the underlying contract:

$$(\$475/\$13{,}500) \times 100 = 3.52\%$$

Exhibit 20.4 Prices of Options on Futures Contracts

Currency	Type	Maturity	Strike Prices					
			900	**950**	**1000**	**1050**	**1100**	**1150**
		May	14.91	9.91	4.93	0.68	0.01	—
Canadian Dollar	**Calls**	**Jun**	14.91	9.93	5.04	1.20	0.08	—
		Sep	14.7	9.91	5.55	2.26	0.63	0.18
CAD100,000		**May**	—	—	0.02	0.77	5.10	10.09
USD cents per CAD	**Puts**	**Jun**	—	0.02	0.14	1.29	5.17	10.09
		Sep	0.11	0.31	0.94	2.63	5.98	10.52
			1000	**1050**	**1100**	**1150**	**1200**	**1250**
		May	13.24	8.24	3.34	0.37	0.02	—
Swiss Franc	**Calls**	**Jun**	13.24	8.27	3.73	0.96	0.17	0.05
		Sep	13.42	8.86	5.04	2.47	1.12	0.59
CHF 125,000		**May**	-	-	0.10	2.13	6.78	10.76
USD Cents per CHF	**Puts**	**Jun**	-	0.04	0.49	2.72	6.92	10.80
		Sep	0.17	0.60	1.76	4.17	7.81	11.27
			14200	**14300**	**14400**	**14500**	**14600**	**14700**
		May	3.85	3.01	2.26	1.6	1.08	0.69
Euro	**Calls**	**Jun**	4.49	3.75	3.09	2.48	1.95	1.49
		Sep	6.04	5.41	4.83	4.29	3.78	3.31
EUR125,000		**May**	0.30	0.46	0.68	1.05	1.53	2.14
USD Cents per EUR	**Puts**	**Jun**	0.94	1.17	1.59	1.93	2.40	2.94
		Sep	2.93	3.30	3.71	4.14	4.66	5.18
			1410	**1460**	**1510**	**1560**	**1610**	**1660**
		May	24.38	19.38	14.38	9.40	4.51	0.83
British Pound	**Calls**	**Jun**	24.38	19.38	14.41	9.50	4.96	1.59
		Sep	24.24	19.42	14.74	10.33	6.46	3.46
GBP62,500		**May**	0.02	0.02	0.01	0.02	0.13	1.45
USD Cents per GBP	**Puts**	**Jun**	0.01	0.01	0.04	0.13	0.53	2.21
		Sep	0.16	0.33	0.64	1.22	2.33	4.32
			1120	**1170**	**1220**	**1270**	**1320**	**1370**
		May	10.32	5.38	1.26	0.16	0.03	0.01
Japanese Yen	**Calls**	**Jun**	10.38	5.65	2.07	0.59	0.18	0.06
		Sep	10.97	6.93	3.95	2.11	1.13	0.62
JPY12,500,000		**May**	0.01	0.06	0.94	4.84	9.71	14.69
USD Cents per 100 JPY	**Puts**	**Jun**	0.07	0.34	1.75	5.26	9.85	14.73
		Sep	0.59	1.54	3.54	6.68	10.69	15.17

Note: All contracts are traded at the CME Group. Data are from Thomson Reuters and the Wall Street Journal Market Data Group.

Currency Options at the CME Group

At the CME Group, the buyer of an option is entitled to the right to buy (for a call) or to sell (for a put) the corresponding currency futures contract. Consequently, the contract sizes and expiration months follow those of the futures contracts. Trading closes on the Friday immediately preceding the third Wednesday of the contract month. Exhibit 20.4 contains examples of options quotes from the CME Group from Thursday, April 21, 2011.

Options Quotes

In Exhibit 20.4, the first column identifies the currency, the contract size, and the units in which option premiums are expressed. For example, the British pound contract size is

£62,500, and prices of the options are quoted in U.S. cents per pound. The option prices for most other currencies are also quoted in cents per unit. The exception is the Japanese yen, where the units are cents per 100 yen. The quotations for the strike prices are unusual, and the user should be aware of current futures prices to ensure a correct interpretation of the units. Most currencies, such as the euro, are quoted in 1/100 cent per unit, as the first euro strike price is 14,200, which corresponds to an exchange rate of $1.42/€. But the strike prices for the pound are quoted in 1/10 cent per pound, as 1,510 corresponds to an exchange rate of $1.51/£, and the strike prices for the yen are in 1/1,000 cent per yen, as 1,270 corresponds to an exchange rate of $0.01270/¥.

Each column related to a contract price provides the strike price in the first row followed by three rows of call prices and three rows of put prices. The three rows refer to different expiration months. The May contract is linked to the May futures contract. To check your understanding of the information provided in Exhibit 20.4, let's consider the purchase of a yen put option contract because the units are a little tricky.

Example 20.5 A Yen Put Option Against Dollars

Consider a Japanese yen put option contract with a strike price of 1,270 ($0.01270/¥) and a maturity of June, which costs 5.26 U.S. cents per 100 yen. If we want to express the strike price in dollars per yen, we must first divide by 100 to convert from cents per 100 yen to cents per yen, and then we must divide by 100 again to convert from cents per yen to dollars per yen. Hence, the cost of the option goes from 5.26 cents per 100 yen to 0.0526 cents per yen, or $0.000526/¥. Consequently, the buyer of the contract would pay

$$(\$0.000526/¥) \times ¥12{,}500{,}000 = \$6{,}575$$

to the seller of the contract at the initiation of the deal. Because the contract is an American-style option, the buyer of the contract would have the right, but not the obligation, to sell ¥12,500,000, or one futures contract, at the strike price of $0.01270/¥ in the futures markets, and the seller would be obligated to purchase the yen futures contract at that price at any time before the June maturity.

Exchange-Listed Currency Warrants

Longer-maturity foreign currency options, called **currency warrants**, are sometimes issued by major corporations or investment banks and are actively traded on exchanges such as the American Stock Exchange, the London Stock Exchange, and the Australian Stock Exchange. Corporate issuers include AT&T Credit Corp., Deutsche Bank, Ford Motor Credit Co., Goldman Sachs, General Electric Credit Corp., the Macquarie Bank Ltd., the Student Loan Marketing Corp. (Sallie Mae), Société Générale, and Xerox Credit Corp. Maturities often exceed 1 year.

Currency warrants allow retail investors and small corporations that are too small to participate in the OTC markets to purchase long-term currency options. In most cases, the original issuer should not be viewed as bearing the implied currency risk. Instead, the issuer is probably hedging in the bank-dominated OTC market. The issuers are effectively buying foreign exchange options at wholesale prices and selling options to the public at a retail price. A currency warrant is generally cash settled, with the payoff clearly explained in the prospectus. Let's look at an example.

Example 20.6 Macquarie Put Warrant

Consider an Australian dollar put warrant against the U.S. dollar issued by Macquarie Bank with a maturity date of December 15, 2010, that traded on the Australian Stock Exchange. The warrant was characterized by a strike price of \$0.90/AUD and a multiplier of AUD10. The payoff to the put warrant was specified as

$$\max\left[0, \frac{\text{Strike price} - \text{Spot rate}}{\text{Spot rate}}\right] \times \text{Multiplier}$$

For example, suppose the spot exchange rate was \$0.85/AUD at maturity. Then, the settlement value for one warrant would have been

$$\frac{(\$0.90/\text{AUD}) - (\$0.85/\text{AUD})}{(\$0.85/\text{AUD})} \times \text{AUD}10 = \$0.59$$

Note that, as is true with exchange-traded options, an investor can close out his position at any point by selling the warrant back into the market. Since the actual spot exchange rate at maturity was \$1.0233/AUD, the holder of the warrant at maturity received no payoff.

20.4 The Use of Options in Risk Management

Now that you understand the basics of foreign currency options, we can examine how they can be used to manage foreign exchange risk. The classic use of a foreign currency option contract as a hedging device arises in a bidding situation.

A Bidding Situation at Bagwell Construction

Suppose that Bagwell Construction, a U.S. company, wants to bid on the construction of a new office building in Tokyo. The Japanese developer has instructed all interested parties to submit their yen-denominated bids by June 30. Because the bids are complex contracts involving many more parameters than just the overall yen price of the contract, it will take the Japanese developer a month to evaluate the bids, and the winner will not be announced until July 31.

Bagwell management has determined that it can do the construction in Tokyo for \$80,000,000, which will be paid out more or less evenly over the course of a year. If the firm gets the contract, it will receive yen revenue from the Japanese developer in five equal installments. There will be an initial yen payment on July 31, followed by four quarterly installments.

The Transaction Risk

By bidding a fixed amount of yen to do this project, Bagwell Construction incurs transaction foreign exchange risk. If Bagwell gets the project and the yen weakens relative to the dollar, the contractual yen revenue will purchase fewer dollars in the future. Notice that as soon as Bagwell bids on the contract, it acquires a transaction exposure. If the firm does nothing to hedge its contingent yen asset exposure during the time that the contracts are being evaluated and the yen weakens relative to the dollar, Bagwell's entire dollar profit could be eliminated before it even begins construction. If its strategy is to get the contract and then hedge, it could be too late.

The Problem with a Forward Hedge

Can Bagwell Construction hedge this risk with a forward contract? If Bagwell sells yen forward, it acquires an uncontingent yen liability. No matter what happens in 30 days, Bagwell will have to sell a specific amount of yen to the bank. Everything will be fine if Bagwell gets the contract. But what would happen if Bagwell sells yen forward and then fails to win the construction contract?

If the company does not get the construction job, it will still have to buy yen to fulfill the uncontingent commitment of the forward contract. If the yen strengthens such that the dollar price of yen in the spot market is higher than the contractual forward price, Bagwell will lose money because it will cost more dollars in the spot market to buy the yen to be delivered on the forward contact than the amount of dollars that the company will receive from the bank. Hence, if the yen strengthens versus the dollar, Bagwell will lose money, possibly a lot of money.

The Options Solution

Foreign exchange options provide a much better solution to Bagwell's problem of hedging in June prior to the resolution of the contract because options provide the purchaser with a contingent claim. How would an option contract work, and which option should be used?

Because Bagwell ultimately wants to sell the yen it will be paid if it gets the contract, the company should hedge by buying a yen put against the dollar. The yen put gives the buyer the right, but not the obligation, to sell yen for dollars at the strike price. Then, if Bagwell gets the contract and the yen has weakened relative to the dollar, the loss of value on the construction contract is offset by a gain in the value of the yen put. The company can sell yen from the construction contract at the exercise price, which is higher than the dollar price of yen in the spot market.

If Bagwell does not win the contract, the value of the yen put is the maximum that the firm can lose. But if at the maturity of the option, the yen has weakened relative to the dollar, the right to sell yen at a high dollar price will be valuable. Bagwell will consequently be able to recoup some of the premium that was initially paid for the option. Purchasing the option thus provides insurance against transaction risk.

Using Options to Hedge Transaction Risk

We now turn to the use of options in managing transaction exchange risk. While forwards and futures can be used, options allow the firm to hedge while retaining some of the upside potential from favorable exchange rate changes. Our next example considers an exporting situation.

Example 20.7 Exporting Pharmaceutical Products from the United States to the United Kingdom

On Friday, October 1, 2010, Pfimerc, an exporter of pharmaceutical products from the United States to the United Kingdom, knew it had an account receivable of £500,000 due on Friday, March 19, 2011. The following data were available:

Spot rate (U.S. cents per British pound): 158.34
170-day forward rate (U.S. cents per British pound): 158.05
U.S. dollar 170-day interest rate: 0.20% p.a.
British pound 170-day interest rate: 0.40% p.a.
Option data for March contracts in ¢/£:

Strike	Call Prices	Put Prices
158	5.00	4.81
159	4.52	5.33
160	4.08	5.89

Pfimerc wanted to understand how it might hedge this transaction using foreign currency options. The first thing to determine is which type of option provides a hedge. Because Pfimerc would be receiving British pounds, the transaction risk is that the pound weakens relative to the dollar. If the company does not hedge, it could experience a large loss when it sells the £500,000 in the spot market in March. The appropriate option hedge gives Pfimerc the right, but not the obligation, to sell pounds in 170 days at a contractual strike price of dollars per pound—a European pound put option.

Let's work with a strike price of 158¢/£, which costs 4.81¢/£ or \$0.0481/£. Because Pfimerc wants to sell £500,000 in the future, today it must pay

$$£500{,}000 \times (\$0.0481/£) = \$24{,}050$$

If in March, the dollar value of the pound falls below the strike price of \$1.58/£, Pfimerc will exercise the option to sell £500,000 at that price. Consequently, the minimum March revenue that Pfimerc will receive is

$$£500{,}000 \times \$1.58/£ = \$790{,}000, \text{ if } S(t{+}170) \leq \$1.58/£$$

When the future spot rate exceeds the strike price, the company will sell its pounds in the future spot market, and its revenue will be

$$£500{,}000 \times S(t{+}170) > \$790{,}000, \text{ if } S(t{+}170) > \$1.58/£$$

Whether Pfimerc exercises the option or not, if it hedges with put options, it must subtract the March value of the cost of the puts that was paid in October from its March revenue to get a net revenue figure. This opportunity cost of purchasing the option is therefore

$$[\$24{,}050 \times (1 + i(\$)] = [\$24{,}050 \times (1.00094)] = \$24{,}073$$

where the interest factor is $(0.2/100)(170/360) = 0.00094$. Hence, the minimum net revenue that Pfimerc receives in March if it hedges with puts is

$$\$790{,}000 - \$24{,}073 = \$765{,}927$$

On a cents-per-pound basis, the March cost of the put option is

$$(4.81¢/£) \times (1.00094) = 4.82¢/£$$

Exhibit 20.5 summarizes the transaction risk exposure related to various strategies for selling British pounds. The horizontal axis shows possible realizations of future spot exchange rates expressed in U.S. cents per pound. The vertical axis measures the net revenue Pfimerc receives (in cents per pound), and the three different lines represent its net revenues depending on the realizations of the future exchange rate.

The 45-degree line represents the unhedged strategy. If Pfimerc chooses not to hedge, it sells pounds for dollars in the future spot market, and its revenue increases one for one with pound appreciation. But, its revenue also decreases one for one with any pound depreciation. Pfimerc's risk of loss is therefore unlimited.

The horizontal line in Exhibit 20.5 represents the strategy of hedging with a forward contract. If Pfimerc sells pounds forward at \$1.5805/£, its March revenue is

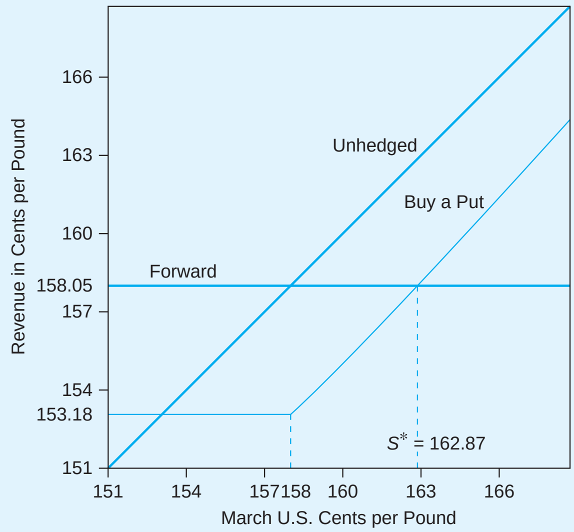
$$(\$1.5805/£) \times £500{,}000 = \$790{,}250$$

On a cents-per-pound basis, Pfimerc's revenue will equal the forward rate of 158.05¢/£ no matter what future spot exchange rate is realized.

The kinked line in Exhibit 20.5 represents the net revenue from the strategy of buying the 158 March pound put. The minimum net revenue is

$$158¢/£ - 4.82¢/£ = 153.18¢/£$$

Exhibit 20.5 Hedging Pound Revenues



Notes: The horizontal axis presents different possible future exchange rates. The vertical axis represents the revenue in cents per pound from three different strategies. The horizontal line reflects the revenue implied by a forward contract, which is not dependent on the future exchange rate. The upward-sloping 45-degree line represents the unhedged strategy. The revenue equals the future exchange rate. The "hockey stick" line represents the revenue from hedging the receivable by buying a pound put option.

This occurs when Pfimerc exercises its puts—that is, when the future spot rate is less than or equal to 158¢/£. The option hedge provides a floor on Pfimerc's revenue while allowing it to participate in any strengthening of the pound relative to the dollar.

Notice also that the net revenue from the option hedge is below the net revenue from the forward hedge when the exchange rate in the future is below a certain exchange rate, denoted S^* in Exhibit 20.5. If the future spot rate is greater than S^*, the net revenue from the option hedge exceeds the revenue from the forward hedge. This is an example of no-free-lunch economics. If the option hedge puts a floor on your net revenue, but it allows you to participate in a possible strengthening of the pound, which increases your net revenue, the floor must be below the forward rate. Otherwise, the option strategy would strictly dominate the forward strategy.

We can determine the value of S^* by equating the two net revenues. The net revenue from the option hedge is $S^* - 4.82¢/£$, and the revenue from the forward hedge is 158.05¢/£. Therefore, we find that S^* is

$$S^* - 4.82¢/£ = 158.05¢/£$$
$$S^* = 162.87¢/£$$

Because the current spot rate is 158.34¢/£, the pound must strengthen relative to the dollar by 2.86%—that is, to 162.87¢/£—before hedging with puts provides a higher net revenue than the forward hedge.

So should Pfimerc use the option strategy or the forward hedge strategy? To decide, the company must calculate the probability that the exchange rate in the future will exceed S^*(¢/£). We discuss how this question is answered later in the chapter.

Example 20.8 Importing Watches to the United States from Switzerland

Consider the case of an importer who must pay in the exporter's currency. Here, the importer will use call options on the exporter's currency to hedge.

Suppose it is Thursday, September 16, and Orlodge, an importer of Swiss watches to the United States, has an account payable of CHF750,000 due on Wednesday, December 15. The following data are available:

Spot rate: 71.42¢/CHF
90-day forward rate: 71.14¢/CHF
U.S. dollar 90-day interest rate: 3.75% p.a.
Swiss franc 90-day interest rate: 5.33% p.a.
Option data for December contracts (¢/CHF):

Strike	Call	Put
70	2.55	1.42
72	1.55	2.40

To hedge this transaction using foreign currency options, Orlodge must first determine the type of option that provides a hedge. Because Orlodge will be paying Swiss francs in 90 days, the transaction risk is that the Swiss franc will strengthen versus the dollar, which increases the cost of the CHF750,000. To hedge, Orlodge should buy the option that gives it the right, but not the obligation, to buy Swiss francs in 90 days at a strike price of dollars per Swiss franc. This is a European Swiss franc call option.

Let's work with the December Swiss franc call option with a strike price of 72¢/CHF. The cost per unit of this contract is 1.55¢/CHF, or \$0.0155/CHF. As the buyer of the contracts, Orlodge must pay today

$$\text{CHF}750{,}000 \times \$0.0155/\text{CHF} = \$11{,}625$$

If, at maturity in December, the dollar value of the Swiss franc is greater than or equal to the strike price of \$0.7200/CHF, Orlodge will exercise its option to buy CHF750,000 at that price. Consequently, Orlodge's maximum payment is

$$\text{CHF}750{,}000 \times \$0.7200/\text{CHF} = \$540{,}000, \text{ if } S(t{+}88) \geq \$0.7200/\text{CHF}$$

At all exchange rates less than \$0.7200/CHF, Orlodge will buy francs in the spot market, and its cost will be

$$\text{CHF}750{,}000 \times S(t{+}88) < \$540{,}000, \text{ if } S(t{+}88) < \$0.7200/\text{CHF}$$

Whether Orlodge exercises its option or not, if it hedges with call options, it must add the December value of the September cost of the call options to the December cost of the Swiss francs to get a total cost figure. This opportunity cost is

$$\$11{,}625 \times [1 + i(\$)] = \$11{,}625 \times (1.0094) = \$11{,}734$$

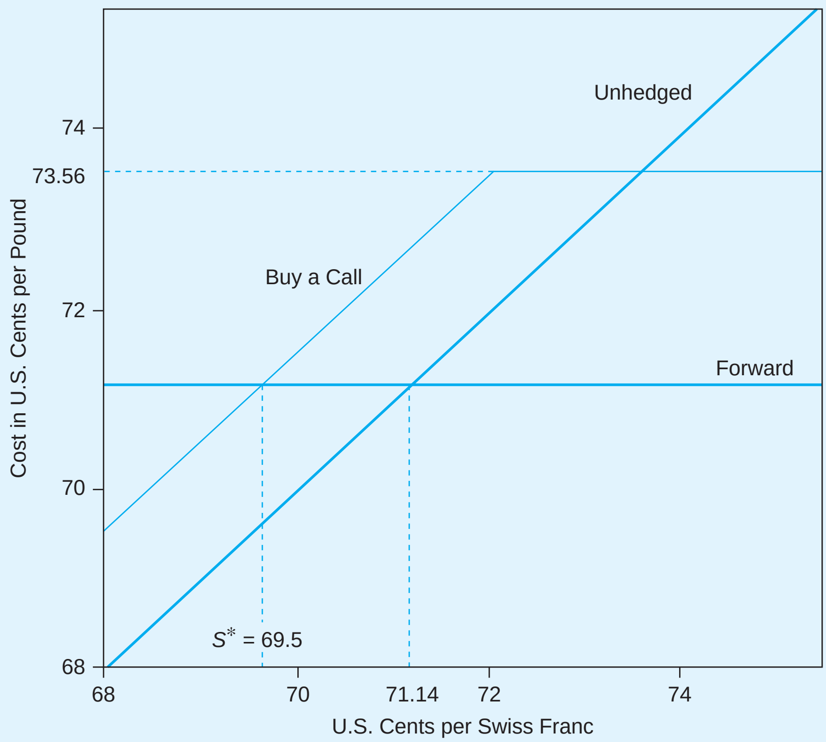
where the interest factor is $(3.75/100)(90/360) = 0.0094$. Hence, Orlodge's maximum December total cost if it hedges with call options is

$$\$540{,}000 + \$11{,}734 = \$551{,}734$$

In cents per Swiss franc, the December cost of the call option is

$$(1.55¢/\text{CHF}) \times (1.0094) = 1.56¢/\text{CHF}$$

Exhibit 20.6 Hedging Swiss Franc Costs



Notes: The horizontal axis presents different possible future exchange rates. The vertical axis represents the costs in cents per Swiss franc from three different strategies. The horizontal line reflects the cost implied by a forward contract, which is not dependent on the future exchange rate. The upward-sloping 45-degree line represents the unhedged strategy: The cost equals the future exchange rate. The inverted "hockey stick" line represents the cost from hedging the payable by buying a call option.

Exhibit 20.6 has possible December values of the exchange rate in cents per Swiss franc on the horizontal axis and the cost in cents per Swiss franc on the vertical axis. The different lines now represent the cost of different strategies, depending on the realization of the future exchange rates. As before, the 45-degree line represents the unhedged strategy. If Orlodge chooses not to hedge, it must buy its Swiss francs with dollars in the future spot market. Its cost will increase one for one with any strengthening of the Swiss franc versus the dollar, but its cost will also be lower, one for one, with any weakening of the Swiss franc. Its risk of loss is therefore unlimited.

The horizontal line in Exhibit 20.6 represents hedging with a forward contract. If Orlodge buys Swiss francs forward at \$0.7114/CHF, its December cost is

$$\$0.7114/\text{CHF} \times \text{CHF}750{,}000 = \$533{,}550$$

On a cents-per-franc basis, Orlodge's cost will be 71.14¢/CHF no matter what spot exchange rate is realized in the future.

The kinked line in Exhibit 20.6 represents the total cost of hedging with the 72 December Swiss franc call options. The maximum total cost is

$$72.00¢/\text{CHF} + 1.56¢/\text{CHF} = 73.56¢/\text{CHF}$$

This cost arises when Orlodge exercises its call options—that is, when the future spot rate in December is greater than or equal to 72.00¢/CHF.

The option hedge provides a ceiling on Orlodge's costs, while allowing it to participate in any strengthening of the dollar relative to the Swiss franc, which can reduce the company's costs. Notice also that the total cost from the option hedge is above the total cost from the forward hedge whenever the exchange rate in the future is above S^*. If the future spot rate is less than S^*, the total cost from the option hedge is less than the cost from the forward hedge. This is another example of no-free-lunch economics: If the call option hedge puts a ceiling on your total cost, but it allows you to participate in a possible strengthening of the dollar that can reduce your costs, the ceiling must be above the forward rate.

The value of S^* equates the total costs of the two hedges. The total cost from the option hedge is $[S^* + 1.56¢/\text{CHF}]$, and the cost from the forward hedge is 71.14¢/CHF. Therefore, solving for S^* gives

$$S^* = 71.14¢/\text{CHF} - 1.56¢/\text{CHF} = 69.58¢/\text{CHF}$$

The Swiss franc must weaken by 2.58% relative to the dollar, from 71.42¢/CHF to 69.58¢/CHF, before the call option contract provides a lower cost than the forward hedge.

As Orlodge considers different strategies for dealing with the Swiss franc payable, including alternative option strategies or the forward hedge, the firm should attempt to calculate the probability that the future spot rate will be less than S^*. We will discuss this in the next section, which compares option hedges to the purchase of insurance.

Hedging with Options as Buying Insurance

In the two previous examples, option strategies hedge transaction exchange risks. Here, we consider how hedging with options is analogous to purchasing insurance. Before we do so, we summarize more generally how to hedge foreign currency receivables and payables with forward, futures, and option contracts. Exhibit 20.7 gives an overview of this discussion. It also includes some speculative option strategies that we will discuss later.

Hedging Foreign Currency Risk with Forwards and Options

Exporters who price in foreign currency generate foreign currency revenues. Their appropriate forward hedge is to sell the foreign currency receivable forward. Their appropriate option

Exhibit 20.7 Hedging and Speculating Strategies

	Underlying Transaction	
	Foreign Currency Receivable	**Foreign Currency Payable**
Forward Hedge (or futures hedge)	Sell forward (Go short)	Buy forward (Go long)
Option Hedge	Buy a put Establishes a revenue floor of $K - (1 + i)P$	Buy a call Establishes a cost ceiling of $K + (1 + i)C$
Option Speculation	Sell a call Imposes a revenue ceiling of $K + (1 + i)C$ but allows unlimited risk	Sell a put Imposes a liability floor of $K - (1 + i)P$ but allows unlimited risk

Notes: K is the strike price, C is the call option premium, P is the put option premium, and i is the appropriate deannualized interest rate factor.

hedge is to buy a foreign currency put. The put provides the right, but not the obligation, to sell the foreign currency revenue at the strike price of domestic currency per foreign currency, which establishes a floor on net revenue equal to the strike price minus the future value of the option premium.

If you buy a put option, you are not contractually committed to sell the export revenue through that option. You retain the right to sell the foreign currency in the spot market if the domestic currency value of the foreign currency exceeds the strike price. A strenghtening of the foreign currency allows your net revenue to exceed the floor established by the put option, and if the foreign currency strengthens sufficiently, your net domestic currency revenue from the option hedge can substantially exceed the revenue from the forward hedge. Nevertheless, because some money is paid up front, the net revenue from the option hedge remains less than the revenue that would have been generated if the option contract had not been purchased. Naturally, this can only be known *ex post*—that is, after the realization of future uncertain exchange rates. But, of course, the strategy must be chosen first.

For importers with foreign currency costs, the appropriate forward hedge is to buy the foreign currency forward. The appropriate option hedge is to buy a **foreign currency call option** contract. This gives you the right, but not the obligation, to buy the foreign currency at the strike price, which places a ceiling on your total costs. The ceiling on your costs is the strike price plus the future value of the option premium.

If you buy a call option, you retain the right to buy foreign currency in the spot market if the domestic currency value of the foreign currency is less than the strike price, and if the domestic currency strengthens, your cost falls below the ceiling. If the domestic currency strengthens a lot, the cost from the option hedge can be substantially less than the cost from the forward hedge. But your total cost can never be less than the cost that would have been generated if the option contract had not been purchased. Once again, this can only be known *ex post*, and, unfortunately, you must choose your strategy first.

Options as Insurance Contracts

How are the examples just discussed like insurance policies? Consider the purchase of fire insurance for a home. A homeowner pays annual premiums for insurance that provides a certain amount of coverage in the event of a fire. The quality of the coverage can be varied. The more of the home's value that the homeowner wants to protect, the more costly is the insurance. Expensive insurance completely replaces the home if it is destroyed by fire, and less expensive policies pay some fraction of the loss.

Clearly, the homeowner puts a ceiling on his possible losses by purchasing fire insurance. If there is a fire, the homeowner can repair the home, and the insurance company pays some part of the bill. But, suppose the homeowner lives in the home for 10 years, and no fires occur. *Ex post*, the homeowner will not have needed fire insurance, but he will have paid 10 years of insurance premiums. The homeowner will also have captured the appreciation in the home's value. Nevertheless, the homeowner will not be as well off as he would have been without purchasing the insurance. Of course, this does not mean that purchasing the insurance was a bad idea. It just means that the homeowner did not need the insurance when he lived in the home.

With foreign currency transaction exposures, purchasing the right type of option is like purchasing an insurance policy. Take Example 20.7, in which Pfimerc has a British pound receivable. A weakening of the pound is like a fire because it destroys part of the value of Pfimerc's pound asset. By contracting in advance with an option, some of the value is replaced. That is, if Pfimerc purchases a put option, it places a floor on the dollar value of its pound receivable, even if the pound depreciates. If, on the other hand, the pound strengthens, that is like an appreciation of the value of the home without a fire. Pfimerc ignores the put option and sells its pounds in the spot market. The put option was not needed just like the insurance policy was not needed if there was no fire.

Changing the Quality of the Insurance Policy

Can we carry the fire insurance analogy further? If a homeowner can purchase different qualities of fire insurance at different prices, is there a range of insurance quality when it comes to hedging foreign exchange risk?

Let's first consider hedging a foreign currency receivable with a put option. High-quality insurance in this context means that the floor on our domestic currency revenue is as high as possible. As we discussed, the floor is directly related to the strike price of the put option. The higher the strike price of the option, the less the foreign currency must depreciate before we can exercise the option and cut our losses. Just as insurance that covers more losses is more expensive, put options with higher strike prices are more expensive. We discuss valuation issues in more detail in the next section.

Similarly, high-quality insurance in the context of a foreign currency liability means that we would like to make the ceiling on our cost of the foreign currency as low as possible. This can be accomplished by buying call options with lower strike prices. Again, there is a trade-off because these options will be more expensive. To fully understand this, let's work through a numeric example.

Example 20.9 Purchasing Better, but More Expensive, Insurance

In Example 20.8, Orlodge was importing Swiss watches, and we worked with a December Swiss franc European call option with a strike price of 72¢ per Swiss franc. The cost to hedge the Swiss franc liability was 1.55¢/CHF. Alternatively, we could choose a December call option with a strike price of 70¢/CHF that costs 2.55¢/CHF. This more expensive "insurance" should provide a lower ceiling on the total Swiss franc cost. The trade-off is that the exchange rate, S^*, at which Orlodge has the same cost as the forward hedge is now lower. Hence, the probability of having a lower cost than the forward hedge is smaller because Orlodge gets a lower cost only if the future exchange rate is less than this new S^*.

Exhibit 20.8 presents the cost diagrams for the two option strategies with strike prices of 70¢/CHF and 72¢/CHF. The initial cost of the insurance from the call option with the lower strike price is

$$\text{CHF}750{,}000 \times (\$0.0255/\text{CHF}) = \$19{,}125$$

compared to the \$11,625 in Example 20.8. At maturity in December, if the dollar value of the Swiss franc is greater than or equal to the strike price of \$0.70/CHF, Orlodge will exercise its option to buy CHF750,000 at that price. Consequently, the maximum that Orlodge will pay in December is

$$\text{CHF}750{,}000 \times \$0.70/\text{CHF} = \$525{,}000, \text{ if } S(t{+}88) \geq \$0.70/\text{CHF}$$

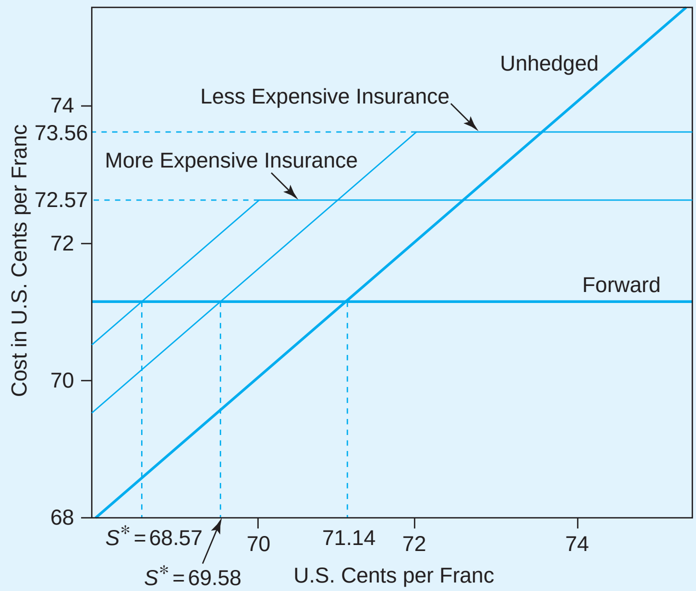
At all exchange rates less than \$0.70/CHF, Orlodge will buy Swiss francs in the spot market, and its cost will be

$$\text{CHF}750{,}000 \times S(t{+}88) < \$525{,}000, \text{ if } S(t{+}88) < \$0.70/\text{CHF}$$

Of course, Orlodge must add the December value of the cost of the call options that was paid in September to the December cost of the Swiss francs to get a total cost figure. This opportunity cost is

$$[\$19{,}125 \times 1.0094] = \$19{,}305$$

Exhibit 20.8 Alternative Option Hedges



Notes: The horizontal axis represents the future exchange rate in cents per Swiss franc. The vertical axis represents the cost in cents per Swiss franc of various strategies for dealing with a Swiss franc liability. The horizontal line shows that a forward hedge locks in a cost per Swiss franc of 71.14 cents. The 45-degree line represents the unhedged strategy, and the two inverted "hockey stick" lines represent the *ex post* costs of two option strategies, struck at different strike prices.

where the interest factor is $(3.75/100)(90/360) = 0.0094$. Hence, the maximum total cost that Orlodge will pay in December if it hedges with call options is

$$\$525{,}000 + \$19{,}305 = \$544{,}305$$

In Example 20.8, the corresponding figure is \$551,734. Hence, Orlodge has improved the quality of its insurance because its total cost is now lower in the bad states of the world in which the dollar weakens relative to the Swiss franc.

On a cents-per-franc basis, the December cost of the call option with a strike price of 70¢/CHF is

$$2.55¢/\text{CHF} \times 1.0094 = 2.57¢/\text{CHF}$$

Hence, the total cost of the liability per unit of foreign currency is, at most,

$$70¢/\text{CHF} + 2.57¢/\text{CHF} = 72.57¢/\text{CHF}$$

We can again determine the value of S^* that equates the cost of the option hedge to the cost of the forward hedge. The total cost from the option hedge is $S^* + 2.57¢/\text{CHF}$, and the cost from the forward hedge is 71.14¢/CHF. Solving for S^* gives

$$S^* = 71.14¢/\text{CHF} - 2.57¢/\text{CHF} = 68.57¢/\text{CHF}$$

This is less than the S^* of 69.58¢/CHF in Example 20.8. With more expensive insurance, more strengthening of the dollar relative to the Swiss franc must occur before Orlodge's cost is lower than the cost of the forward hedge. Because the current spot rate is 71.42¢/CHF, the Swiss franc must weaken by 3.99%, to 68.57¢/CHF, before the call option contract with a strike price of 70¢/CHF provides a lower total cost than the forward hedge.

Speculating with Options

Examples 20.7 and 20.8 discuss hedging transaction exchange risk with options. Choosing the right strategy in these examples is tantamount to purchasing insurance. Sometimes, firms think that this insurance is too expensive. If it is, a firm can profit from a speculative strategy as long as the realized future exchange rate remains in certain regions. That is, rather than purchase insurance, you can use the option markets to sell insurance.

If purchasing a put provides insurance when you have a foreign currency receivable, then selling a call allows you to sell the foreign currency, either to the purchaser of the call option or in the spot market, and your revenue is enhanced by the option premium. Of course, you are now selling insurance to someone who may want to exercise the option.

Similarly, if purchasing a call seems too expensive when hedging a foreign currency liability, you might want to write a put. The put obligates you to buy the foreign currency at the strike price when the buyer of the put exercises that option to sell foreign currency to you. Once again, though, the option premium provides you with revenue that lowers the effective cost of your foreign currency liability.

While we illustrate these strategies, you should understand that speculating does not protect the firm's revenue from potential losses or its cost from potential increases due to exchange rate changes. Some of the large foreign exchange losses experienced by firms in the recent financial crisis arose because they were following complex versions of these speculative strategies, either through ignorance of the possible losses or an assessment that the *ex ante* risk was worth taking. We come back to this issue in Section 20.5.

Speculating on Foreign Currency Receivables

Let's illustrate these speculative strategies with the foreign currency receivable in Example 20.7. Suppose Pfimerc is scheduled to receive £500,000 in 170 days. The pound put option provides the hedge: It gives Pfimerc the right, but not the obligation, to sell pounds at a contractual strike price of dollars per pound. But suppose this put option seems expensive. Would a different option strategy allow Pfimerc to sell pounds for dollars and have the potential to generate more dollar revenue?

Pfimerc could achieve this objective by selling someone the right, but not the obligation, to buy pounds from it in exchange for dollars. This option describes a pound call option against the dollar. Because Pfimerc knows the date on which it wants to sell pounds and the amount of pounds it wants to sell, it could sell someone a European pound call option against the dollar with 170 days until maturity. When Pfimerc sells the pound call option, it generates dollar revenue in September, and this revenue enhances its dollar return in the future.

This strategy is speculative, though, because Pfimerc loses protection against downside risk. If the pound weakens substantially relative to the dollar, the purchaser of the pound call option from Pfimerc will find it to be worthless. Pfimerc will be forced to sell its pounds in the spot market precisely when the dollar value of those pounds is low. Also, its ability to participate in a strengthening of the pound versus the dollar is limited.

Suppose that at maturity the dollar–pound spot rate is above the exercise price of the call option contract. The purchaser of Pfimerc's call option will consequently want to buy pounds at the exercise price. Pfimerc will therefore have to sell the pounds at the exercise price. The company will then miss participating in any further strengthening of the pound relative to the dollar. Nevertheless, Pfimerc does take in revenue for selling the call options, and if options are expensive, this revenue can be substantial.

Example 20.10 Speculating on British Pound Receivables

To see how speculating on receivables works with actual data, let's examine the options on British pounds we used before. The March British pound call option with a strike price of 158¢/£ costs 5.00¢/£, or \$0.05/£. If Pfimerc sells the call option in October, it generates revenue of

$$£500{,}000 \times \$0.05/£ = \$25{,}000$$

In March, if the dollar value of the pound is above the strike price of \$1.58/£, Pfimerc will have to sell £500,000 to the option buyer, who will exercise the option to buy pounds at the strike price. Pfimerc's maximum revenue in March will therefore be

$$£500{,}000 \times \$1.58/£ = \$790{,}000, \text{ if } S\,(t{+}32) > \$1.58/£$$

At all exchange rates less than or equal to \$1.58/£, the option Pfimerc sold will be worthless, so Pfimerc will sell its pounds in the spot market instead. Its revenue in March will then be

$$£500{,}000 \times S(t{+}32) \leq \$790{,}000, \text{ if } S\,(t{+}32) \leq \$1.58/£$$

In both cases, though, Pfimerc can add the March value of the October revenue from the option sale to get net revenue. This additional revenue is

$$\$25{,}000 \times (1 + i(\$)) = \$25{,}000 \times 1.00094 = \$25{,}024$$

where the interest factor is $(0.20/100)(170/360) = 0.00094$. Hence, the maximum net revenue that Pfimerc receives in March if it sells the call option is

$$\$790{,}000 + \$25{,}024 = \$815{,}024$$

On a cents-per-pound basis, the additional March revenue is

$$5.0¢/£ \times 1.00094 = 5.01¢/£$$

This is the amount of extra revenue on a cents-per-pound basis that Pfimerc can use to offset any weakening of the pound. To find the future spot exchange rate, $S^*(¢/£)$, at which Pfimerc has the same revenue as the forward rate, we equate the revenue from the two strategies:

$$S^*(¢/£) + 5.01¢/£ = 158.05¢/£$$
$$S^*(¢/£) = 153.04¢/£$$

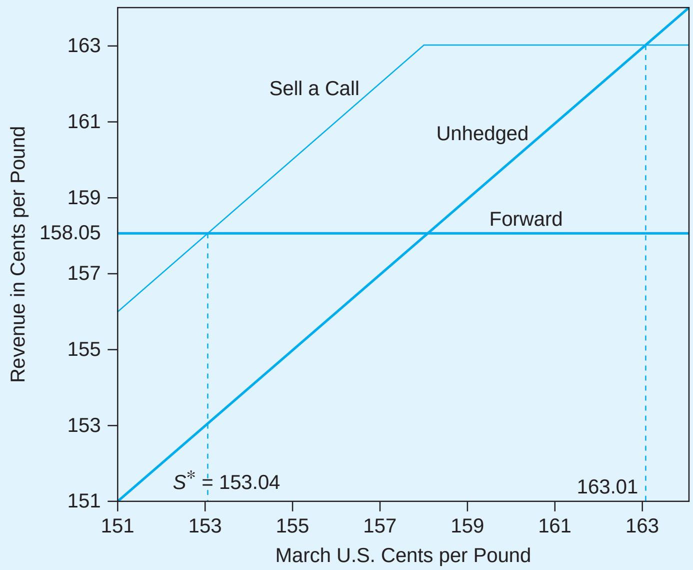
Because the current spot exchange rate is 158.34¢/£, the pound would have to weaken by 3.35% over the next 170 days before this strategy generated lower revenue than the forward hedge. Exhibit 20.9 illustrates the revenue payoff for this speculative strategy.

Notice that there is a range of values of future spot rates over which this speculative strategy has the highest net revenue. On a cents-per-pound basis, maximum revenue from selling the option equals the strike price of 158¢/£ plus the 5.01¢/£. Consequently, the spot exchange rate in the future must be

$$158¢/£ + 5.01¢/£ = 163.01¢/£$$

before the unhedged strategy provides more revenue *ex post* than the speculative option strategy. This requires an appreciation of the pound of 2.95% over the course of 170 days. If you think that the volatility of the exchange rate is not very large, the probability of it reaching this value may not be very large.

Exhibit 20.9 Speculating with Pound Revenue



Notes: The horizontal axis represents the future exchange rate in cents per pound. The vertical axis represents the revenue in cents per pound of various strategies for selling a pound asset. The horizontal line shows that a forward hedge locks in revenue of 158.05 cents. The 45-degree line represents the unhedged strategy, and the inverted "hockey stick" line represents the *ex post* revenue from the strategy of selling a call option with a strike price of 158.

Speculating on Foreign Currency Liabilities

Exhibit 20.7 summarizes how the speculative strategies work. In the case of a foreign currency liability, you must buy foreign currency. Selling someone a **foreign currency put option** forces you to buy the foreign currency at the strike price when the buyer of the option finds it advantageous to sell foreign currency to you—that is, when the exchange rate of domestic currency per foreign currency is lower than the strike price. If the exchange rate ends up higher than the strike price, the option expires worthless, and you must buy the foreign currency in the spot market, exactly when it is relatively expensive. However, whatever happens, writing the option yields revenue, and this strategy may be advantageous when the exchange rate is not anticipated to move very far from its current value.

Options Valuation

We saw that the buyer of an option pays a premium to the seller of the option. How expensive is this type of contract? The purpose of this section is to give you an intuitive idea about how options are valued. The actual formal valuation of options is discussed in the appendix to this chapter because it is quite mathematically complex.[4]

The Intrinsic Value of an Option

Recall that the intrinsic value of an American option is the return, or revenue, generated from the immediate exercise of the option. Intrinsic value is another way of describing whether an

[4]An Excel spreadsheet that performs the calculations can be downloaded from Professor Hodrick's Columbia Business School Web site. Values of foreign currency options are usually discussed in terms of the Garman-Kolhagen (see Garman and Kolhagen, 1983) model, an extension of the famous Black-Scholes (see Black and Scholes, 1973) model.

option is in the money, at the money, or out of the money. So, if K is the strike price of a euro call option against the dollar, and S is the current spot exchange rate, both expressed in $\$/€$, then

$$\text{Intrinsic value of the euro call} = S - K, \text{ if } S > K$$
$$\text{Intrinsic value of the euro call} = 0, \text{ if } S \leq K$$

Because the buyer of the call option must pay the seller of the option for the right to exercise it, the option's price (or its value) must be at least as great as the intrinsic value of the option. The intrinsic value of a call is positive if the strike price is below the current spot exchange rate because the buyer of the option could exercise the right to buy pounds at K and then sell euros in the spot market for the higher price S. If the strike price is higher than the spot rate, immediately exercising the option would result in a loss of money, so the intrinsic value of the option is 0. The option is out of the money.

For an American-style euro put option, we have the following relationships:

$$\text{Intrinsic value of the euro put} = K - S, \text{ if } S < K$$
$$\text{Intrinsic value of the euro put} = 0, \text{ if } S \geq K$$

Once again, because the buyer of the put option must pay the seller of the option for the right to exercise it, the option's price (or its value) must be at least as great as the intrinsic value of the option. The intrinsic value of a put is positive if the put's strike price is greater than the current exchange rate because the buyer of the option could exercise her right to sell euros at K, having bought euros in the spot market for the lower price S. If the strike price is lower than the spot rate, immediately exercising the option would result in a loss of money. Therefore, the option's intrinsic value is 0. The option is out of the money.

The Time Value of an Option

The **time value** of an option is the current price or value of the option minus its intrinsic value:

$$\text{Time value of an option} = \text{Option price} - \text{Intrinsic value}$$

To understand what creates time value, think about a European call option—that is, an option that can only be exercised at maturity. To be concrete, let's think of a euro call option against dollars with a maturity of 90 days.

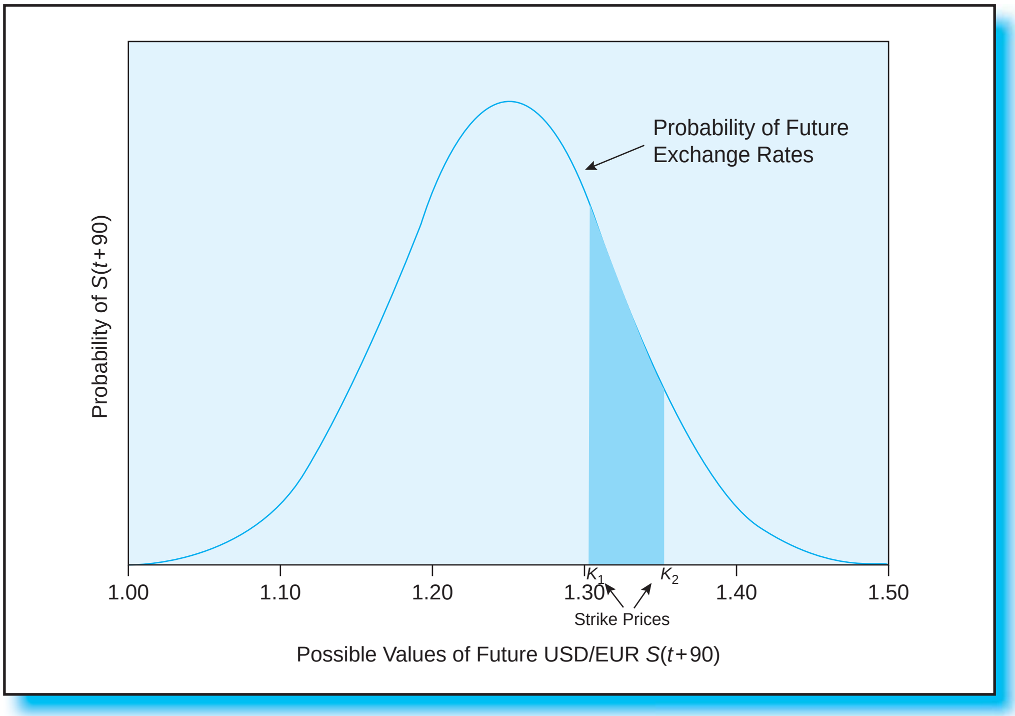
When we introduced forward contracts in Chapter 3, we discussed the probability distribution of future spot exchange rates. Based on our information today, we do not know exactly what the exchange rate of dollars per euros will be in 90 days. Hence, we express our ignorance with a probability distribution, as in Exhibit 20.10. Exhibit 20.10 indicates that the expected value of the dollar–euro rate is $\$1.25/€$ and that values between $\$1.10/€$ and $\$1.40/€$ are fairly likely, while values less than $\$1.00/€$ and greater than $\$1.50/€$ are possible but unlikely to happen.

Exhibit 20.10 has two strike prices, K_1 and K_2. Focus first on K_1. If you buy a European call option on the euro with strike price K_1, you have the right to buy euros at K_1 and then sell the euros in the spot market. You will only do so if the future spot exchange rate of dollars per euro is greater than the exercise price of the option in which case your dollar revenue is $S(t+90) - K_1$. Hence, we can write that for a European option, the euro call option price at time t, $C(t)$, is

$$C(t) = \text{Value at time } t \text{ of } \max[0, S(t+90) - K_1]$$

To determine the value of an option, we must take the present value of the option payoff at the maturity of the contract, which is a non-trivial problem. At this point, it is sufficient to simply understand the intuition of what gives options value.

Exhibit 20.10 Different Probability Distributions of Future USD/EUR



Increasing the Exercise Price

If we hold constant the maturity date of an option, we hold constant the probability distribution in Exhibit 20.10. Now, let's think about increasing the strike price of the option from K_1 to K_2. What happens to the value of the call option? It should be apparent that increasing the exercise price of a euro call must decrease the value of a call option because it removes possible states of the world over which the contract provides revenue when the strike price is lower.

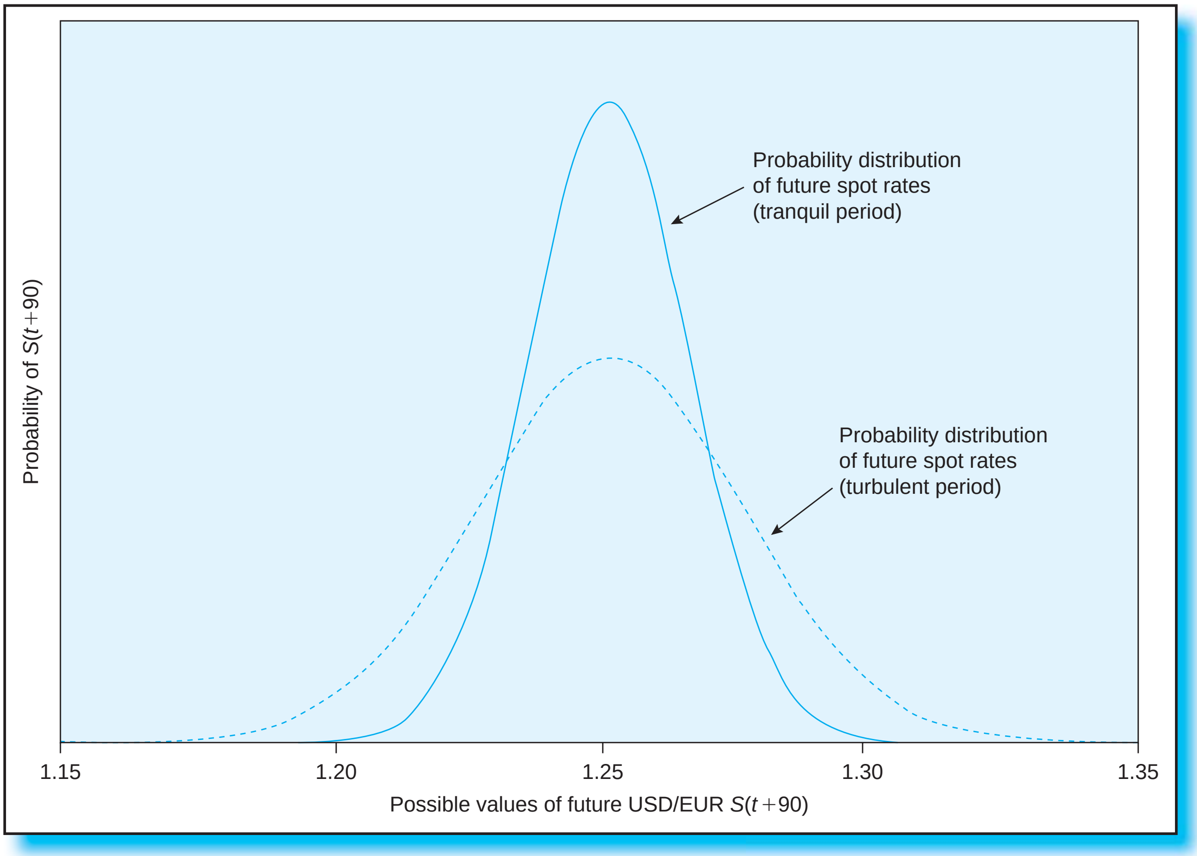
The region of the probability distribution to the right of K_1 gives the probability that the call option with a strike price of K_1 will be exercised. The shaded region contains the additional probability for which the call option with strike price K_1 will be exercised relative to the probability of exercising the option with the higher strike price of K_2. So, when we increase the exercise price from K_1 to K_2, we lose the probability of generating the revenue associated with the shaded region, which makes the option with the strike price K_2 less valuable than the option with the strike price K_1.

A put option provides revenue to the buyer at expiration only if the spot rate in the future is less than the option's exercise price. Hence, increasing the exercise price of a put option must increase the value of a put option because it increases the possible states of the world over which the contract is profitable. We see this in Exhibit 20.10. The probability of exercising the option with a strike price of K_2 is the area of the probability distribution below K_2. The shaded area of Exhibit 20.10 gives the additional probability of exercising an option with strike price K_2 versus one with strike price equal to K_1.

An Increase in the Variance

How does increasing the variance of future exchange rates affect an option's value? Exhibit 20.11 compares two probability distributions—one with a small variance, associated with tranquil periods, and one with a larger variance, associated with turbulent periods. To understand how an increase in variance affects option prices, suppose we place the strike price of

Exhibit 20.11 Different Probability Distributions of Future USD/EUR



a call option at the conditional mean of the two probability distributions in Exhibit 20.11—that is, we choose $K = \$1.25/€$. The increase in the variance of the possible future exchange rate clearly increases the possible range of future exchange rates. But, because the conditional mean is still $1.25/€, the probability that the option will finish in the money is still one-half because one-half of the probability distribution remains above the strike price. However, if the option does finish in the money, the distribution with the larger variance yields possibly larger payoffs, and the option will cost more. A symmetrical argument can be applied to a put option. (Can you explain how?) Therefore, an increase in the variance of future possible exchange rates increases both call and put option values.

Increasing the Time to Expiration

How does increasing the time to expiration affect an option's value? Here, it is important to distinguish clearly between American-style and European-style options. For American options, the effect is unambiguous: Increasing the time to maturity always increases an option's value because it increases the uncertainty of the spot exchange rate at maturity. When this effect is combined with the fact that the holder of a 6-month option can always treat the option as a 3-month option, we clearly see that the additional 3 months of maturity cannot hurt the payoff to the holder of the option as long as the holder of the option can exercise it early.

For European options, the situation is not so simple. Although the effect of an increase in time to maturity is technically ambiguous, in most situations, the effect of the increased uncertainty of the spot exchange rate at maturity dominates, and option prices increase. Nevertheless, this is not always true because it is possible for a European option that is currently in the money to lose value as time evolves. You would like to be able to exercise the option to lock in the revenue now, but you cannot do so prior to maturity.

Put–Call Parity for Foreign Currency Options

The fact that you can hedge and speculate with options suggests that there should be a link between the prices of the put and call options for a given strike price and the forward foreign exchange rate. Because money changes hands at the beginning of option transactions as well as at the end, the interest rate must enter the relationship as well. **Put–call parity** is the fundamental no arbitrage relationship that links the common strike price of domestic currency per unit of foreign currency, the domestic currency prices of European-style put and call options at that strike price, and the domestic currency interest rate. How can we derive this no arbitrage relationship?

Let's work with dollar–euro exchange rates. One way to unconditionally sell euros for dollars (that is, you sell euros for all realizations of the future exchange rate) is to sell at the rate of F. A synthetic way to sell euros unconditionally in the future involves two option transactions. If you purchase a euro put option against the dollar with a strike price of K, you will exercise the option whenever the dollar–euro spot exchange rate at maturity, $S(T)$, is less than K. Let your dollar cost of purchasing the put option be P. If you sell or write a call option with the same strike price of K, you give someone else the right to purchase euros from you at that strike price. She will exercise her option whenever $S(T)$ is greater than or equal to K. You will charge the purchaser of the call option C dollars today.

When both of these option transactions are done simultaneously, you will sell euros in the future at the strike price, K, no matter what happens to the future spot rate. That is, at maturity, you will unconditionally sell euros for dollars at the strike price, K, but your dollar revenue will be enhanced by the future value of the difference between the revenue from selling the call option and the cost of buying the put option, which is $(C - P)[1 + i(\$)]$. Therefore, the two option transactions create a **synthetic forward contract**, and absence of arbitrage requires that the forward exchange rate must be equal to the strike price adjusted for the future values of the revenue from selling the call minus the cost of purchasing the put. That is, put–call parity requires

$$F = K + (C - P)[1 + i(\$)]$$

If you can purchase euros in the forward market at F dollars per euro, and this price is less than the dollar price at which you can synthetically sell euros forward through the two option transactions just described, you can obviously make money. Such an arbitrage transaction is called a **conversion**.

What if the market's forward price is higher than the synthetic forward price? In this case, traders do what is called a **reversal**: They create a synthetic forward purchase of euros and contract to sell euros in the forward market. The synthetic purchase of euros can be done by buying a euro call option with strike price K, which generates a cost of C today; selling a put option with the same strike price, which brings in revenue of P today; and investing or borrowing the difference. The future profit on a reversal is therefore

$$F - K - (C - P)[1 + i(\$)]$$

When neither conversions nor reversals are profitable, the market prices satisfy put–call parity.

Of course, as with interest rate parity, put–call parity will not be an exact equality because it is difficult to do the required transactions simultaneously, and there are transaction costs. Because such costs are typically small, actual option prices are usually close to those implied by put–call parity.

Example 20.11 Putting Numbers to Put–Call Parity

To illustrate how put–call parity works, let's consider the exchange rates and options that Orlodge was facing in Example 20.8. We'll use the options with a strike price of 70¢/CHF. Note that the call option costs 2.55¢/CHF, which is more than the 1.42¢/CHF cost of the put option. We should expect the call option to cost more than the put because the call option is in the money, whereas the put option is not (the current exchange rate is 71.42¢/CHF). Put–call parity states that one can sell the Swiss franc forward at a predetermined rate in two ways: through a forward contract or through buying a put and writing a call. Recall from Example 20.8 that the forward rate is 71.14¢/CHF. The option strategy yields an effective rate of the strike price plus the net cost or revenue of the two option transactions, adjusted for the time value of money. That is, the synthetic forward rate obtained by buying a put and writing a call is

$$70¢/\text{CHF} + (2.55\,¢/\text{CHF} - 1.42¢/\text{CHF}) \times \left(1 + 0.0375 \times \frac{90}{360}\right) = 71.14¢/\text{CHF}$$

This is the same as the forward rate. Hence, put–call parity holds in these quotes. You can verify that it also holds for options with a 72¢/CHF strike price.

20.5 Combinations of Options and Exotic Options

Corporations and institutional investors are increasingly using options and other derivative instruments to manage their exchange rate and interest rate risks.[5] Hedge funds and other institutional investors also often want to invest in instruments that allow them to express their views about various risks and rewards in currency markets. Consequently, investment banks now design products specifically for the tastes of their clients. Often, such products represent combinations of basic put and call options that lower the cost of managing a particular risk. Options with different payoff patterns and features than the basic options discussed in this chapter are mostly referred to as **exotic options**. Some of the more standard exotic options are discussed briefly in this section.

First, though, a word of caution for the purchasers of these options. How can banks offer exotic options that seem like good deals to clients? Banks will hedge an exotic option position by doing the opposite transaction with some other counterparty or by creating synthetic options by trading the underlying assets to offset their risk. Of course, because you can't get something for nothing, purchasers of exotic options should be aware that the ability of the bank to offer such a contract indicates that the purchaser's distribution of future spot rates is probably somewhat different from the market's implied distribution of future spot rates. For example, in terms of Exhibit 20.11, if option prices seem expensive to you, it may be because the market is pricing options from a distribution with a wider conditional variance than you are using. Of course, your personal distribution of future exchange rates may differ in other ways from the market's distribution, and you may be right. But, you should be careful not to delude yourself into thinking that you are getting a good deal; you need to understand the distribution implied by market prices and the implied payoffs on your contract.

[5]See the discussion of current risk management practices in Chapter 17.

Range Forwards and Cylinder Options

Corporate treasurers often argue that option strategies are expensive. They dislike incurring the upfront cost of option premiums. They also encounter difficulty explaining their hedging expenses to their superiors, especially when the insurance they purchase seems to have been unnecessary after the fact. Financial institutions have proposed several solutions that retain some of the hedging features of options but reduce the upfront costs. One solution, designed in 1985 by the investment bank Salomon Brothers, is a range forward contract. A **range forward contract** allows a company to specify a range of future spot rates over which the firm can sell or buy foreign currency at the future spot rate. When the future spot rate falls outside the range, the firm sells or buys the currency at the limits of the range. For example, if the firm is selling foreign currency, it enters into a contract to sell the currency for dollars within a particular range. This creates a floor on the firm's dollar revenue in case the foreign currency weakens. However, it also creates a ceiling on the firm's dollar revenue in case the foreign currency strengthens. If the firm's treasurer thinks that the foreign currency is unlikely to strengthen, or at least not strengthen very much, she will not believe she is sacrificing any upside potential.

At exchange rates in between the limits of the range forward contract, the firm simply sells its foreign currency at the spot rate in the future. Although the firm gets some upside potential, the firm doesn't need to pay money up front for the range forward contract. Range forward contracts were quickly modified by Citibank and other financial institutions, which developed cylinder options. **Cylinder options** allow buyers to specify a desired trading range and either pay money or possibly receive money up front for entering into the contracts.

Synthesizing Cylinder Options

How can we use our knowledge of call and put options to construct synthetic cylinder options and range forward contracts when we are selling foreign currency in the future? Consider a slight modification to our derivation of put–call parity. Here, we express all exchange rates in dollars per pound, just to be concrete.

Suppose you must buy pounds in the future to pay for some British goods. It is possible to construct cylinder options or a range forward contract that allows you to buy pounds in the future at the spot rate over a particular range but places a ceiling on your costs to provide you with insurance. Unfortunately, you must also agree to have a floor on your costs that prevents you from participating fully in dollar appreciation. The ceiling on your costs is established by purchasing a call option, and the floor is established by selling a put option at a lower strike price.

Let the strike price of the put option that is sold be K_p, and let the strike price of the call option that is purchased be K_c, with $K_p < K_c$. Then, depending on the realization of the future spot rate, you will buy pounds in the following way:

If $S \leq K_p$, you buy pounds at K_p because the put you wrote is exercised.
If $K_p < S < K_c$, you buy pounds at S with no exercise of options.
If $S \geq K_c$, you buy pounds at K_c by exercising your call.

In all cases, the firm has an expense equal to the future value of the call premium, $C(K_c)$, that it purchased, and it has revenue equal to the future value of the put premium, $P(K_P)$, that it sold. Hence, its net revenue is augmented by

$$[P(K_p) - C(K_c)] \times [1 + i(\$)]$$

This additional revenue can be adjusted by changing the strike prices on the options to be either positive, negative, or zero. Because the range forward contract requires no cash flows other than the purchase of the pounds, the strike prices must be set such that $P(K_p) = C(K_c)$. The firm might propose the ceiling on its trading range, which establishes the strike price of the call, and the investment bank then sets the floor of the trading range to correspond to the strike price of a put option with the same value as the call option.

Example 20.12 A Cylinder Option Contract

Let's work with some data to create a synthetic cylinder option for a situation in which you have an inflow of foreign currency. As in Example 20.7, suppose it is October, and Pfimerc has a £500,000 account receivable due in March. The following data are available:

Spot rate (U.S. cents per British pound): 158.34
170-day forward rate (U.S. cents per British pound): 158.05
U.S. dollar 170-day interest rate: 0.20% p.a.
British pound 170-day interest rate: 0.40% p.a.
Option data for March contracts in cents per pound (¢/£):

Strike	Call Prices	Put Prices
158	5.00	4.81
159	4.52	5.33
160	4.08	5.89

In Example 20.7, Pfimerc bought the March put option with a strike price of 158¢/£ at a cost of 4.81¢/£. This established a floor on their revenue. Now, suppose that Pfimerc wants to guarantee itself the right to exchange the £500,000 in the range between \$1.58/£ to \$1.60/£. Pfimerc could purchase the 158 March put option for 4.81¢/£ and sell the 160 call option for 4.08¢/£. The net cost from the two option contracts would be $4.81¢/£ - 4.08¢/£ = 0.73¢/£$, or \$0.0073/£. The future value of this net revenue using the interest rate calculated in Example 20.7 is

$$£500{,}000 \times \$0.0073/£ \times 1.00094 = \$3{,}653$$

With these two transactions, Pfimerc's dollar revenue would range from

$$(£500{,}000 \times \$1.58/£) - \$3{,}653 = \$786{,}347$$

if $S(\$/£) \leq \$1.58/£$ to

$$(£500{,}000 \times \$1.60/£) - \$3{,}653 = \$796{,}347$$

if $S(\$/£) \geq \$1.60/£$. This range of revenues can be compared to the forward contract. If Pfimerc sells pounds forward at \$1.5805/£, its March revenue is

$$\$1.5805/£ \times £500{,}000 = \$790{,}250$$

Other Exotic Options

Average-Rate Options

An **average-rate option**, which is sometimes called an Asian option, is one of the most common exotic options. The payoff on an average-rate call option on one unit of foreign currency with a strike price of K is $\max[0, \overline{S} - K]$, where $\overline{S}$ defines the average exchange rate between the initiation of the contract and the expiration date. To calculate the average exchange rate, the counterparties to the option contract must agree on a source for the data and a way of computing the average. They must decide on a time interval for the observations entering the average, which could be daily, weekly, or monthly, and they must decide whether the average is an arithmetic or geometric average.[6] At the maturity of an average-rate option, the seller of the option settles the contract by delivering the option payoff to the buyer. Because an average of future exchange rates is less volatile than the future spot rate at maturity, average-rate options are less expensive than standard European options.

Barrier Options

A **barrier option** is like a traditional option, with an additional requirement that either activates the option or extinguishes it if the exchange rate passes through a prespecified barrier exchange rate. For example, suppose the current exchange rate is \$1.50/£. A 1-year, up-and-out European put option on the pound with a strike price of \$1.45/£ and a barrier of \$1.53/£ specifies that the holder of the option has the right, but not the obligation, to sell pounds for dollars at \$1.45/£ in 1 year unless the exchange rate crosses the barrier of \$1.53/£ prior to the maturity of the option. If the exchange rate crosses the barrier, the option is worthless. Such an option is desirable for people who have pound receivables because they may think that the put option hedge is not necessary if the pound strengthens during the life of the contract.

Barrier options can be either calls or puts, and there are four essential varieties. In addition to the up-and-out option described earlier, there are up-and-in, down-and-out, and down-and-in options. Each of these options specifies a barrier that either activates the option, in the cases of the up-and-in and down-and-in options, or that extinguishes the option if the barrier is crossed, in the cases of the up-and-out and down-and-out options.

Lookback Options

Suppose you want to assure yourself today that in 1 year, you will have bought foreign exchange at the minimum dollar value that occurs during the coming year. You can actually do this by purchasing a **lookback option**. For example, let S_{min} be the minimum exchange rate (in dollars per foreign currency) realized during the year, and let $S(T)$ be the exchange rate in 1 year. The payoff on the lookback call option is

$$\max[0, S(T) - S_{\text{min}}]$$

Because the minimum exchange rate may occur on the last day, $S(T)$ is at least as big as S_{min}, and the payoff can be written as $S(T) - S_{\text{min}}$. A lookback put option can be defined analogously. It allows you to sell foreign currency at the highest exchange rate of dollars per foreign currency that is realized during the life of the option. Of course, when you transact with a lookback option, you are transacting at the prices that are the most favorable to you. Hence, lookback options are more expensive than traditional call and put options.

Digital Options

The two basic **digital options**, or binary options, are cash-or-nothing and asset-or-nothing options. They can be European or American; they can be structured as a call or a put; and they are mostly cash settled. A European cash-or-nothing digital option pays off a fixed amount of money when it expires in the money and nothing otherwise. For example, suppose you buy a digital call option on the dollar/euro exchange rate with a strike price of \$1.35/€ and a principal of \$1,000,000. If, at expiration, the exchange rate is higher than \$1.35/€, you obtain the \$1,000,000; if not, the payoff is 0. The American equivalent of this digital option pays off \$1,000,000 if the exchange rate reaches the \$1.35/€ level any time before expiration. Obviously, such options are issued only at strike prices that are out of the money. If the payout is specified in euros (for example, €1,000,000), the option is really an asset-or-nothing option because the dollar amount represented by the euro payoff, $S(T) \times$ €1,000,000, is uncertain from the perspective of the U.S. investor.

[6]If there are n observations, the arithmetic average is $(1/n)\sum_{i=1}^{n} S_i$, and the geometric average is $\left(\prod_{i=1}^{n} S_i\right)^{1/n}$, where $\sum$ denotes the summation operator and $\prod$ denotes the product operator.

Binary options are interesting because they are useful building blocks in the creation of complex payoff patterns. For example, an option that pays off a very large amount when the exchange rate is within a certain range (a sort of lottery payoff) can be constructed by buying and selling digital call options with different strike prices.

How KIKOs Can Knock You Out

Before the financial crisis, exporters in many emerging economies witnessed surging export volumes but not corresponding increases in profitability as many emerging-market currencies also strongly appreciated in value relative to the U.S. dollar and other major currencies. In trying to hedge their foreign exchange risk while enhancing their revenues, scores of exporters in emerging markets got badly burned by exotic derivatives during the 2007 to 2010 financial crisis. According to Dodd (2009), possibly 50,000 firms in at least 12 economies suffered derivatives losses estimated to be a staggering $530 billion.

To get an idea of what happened, let's focus on the KIKO contracts that many small and medium-size Korean firms used. The exact details of the contracts differed across countries and firms, but they all shared many features with the KIKO contracts. KIKO stands for kick in, kick out, and the contracts can be understood using a combination of put–call parity, barrier options, and leverage.

Consider the situation of Kumkang Valve, a small Korean exporter of valves that open and close oil and gas pipelines [see Lee (2009) for more on the plight of this company]. As the Korean won strengthened versus the dollar prior to the crisis, Kumkang Valve's dollar revenues became worth less and less in Korean won. Hedging foreign currency receivables could be done using a forward contract to sell dollars for won or by buying a dollar put against the won. The KIKO contract essentially combined the buying of a dollar put against won with the selling of a dollar call against won at the same strike price. We know from put–call parity that, if done for the same dollar amount, this strategy is equivalent to selling dollars forward, which is exactly what Kumkang Valve would need to do to hedge its dollar transaction exchange risk.

However, KIKOs added a few twists. First, the amount involved in the call transaction was double the amount involved in the put. This now places the company at risk if the dollar appreciates, placing the won/dollar exchange rate above the strike price; it will incur losses on the call option it sold on the dollar (while of course, its foreign exchange revenues may increase as the dollar is worth more won). Second, the KIKO contract involved a "kick-out" barrier; when the won/dollar exchange rate reaches a particular value, the gains on the put are "kicked out." This makes the dollar put option less expensive. Analogously, it also involved a "kick-in" option: The losses on the won/dollar exchange rate only kick in after the won/dollar exchange rate rises above a particular value, which in turn reduces the value of the call premium earned by the company. The KIKO contract was structured to have zero premiums at initialization. It typically involved a long series of contracts for multiple maturity months in the future. The zero premium structure probably made the contract an easier sell as no initial costs were paid, and financial accounting rules allowed it to be reported as a hedging transaction. It is likely that many companies felt the exchange rate would never hit the "kick in" loss region, as currency forecasts called for further appreciation of emerging-market currencies.

Unfortunately, these forecasts were wrong. As the global financial crisis really took off in 2008, investors flocked to the U.S. dollar as a safe haven, and many emerging-market currencies, including the Korean won, experienced steep depreciations. The "kick-in" barrier was breached, and the losses for KIKO investors started to mount. Kumkang Valve filed for bankruptcy in September 2008. Dodd (2009) claims that reports of losses on derivatives at many companies roiled the local currency markets, amplifying selling pressures. Because the OTC markets for these exotic derivatives are not transparent, the currency markets were in the dark about the total amounts of the outstanding transactions and the magnitudes of the potential losses. This lack of information may have led to uncertainty, which potentially further depressed currency values and, in turn, caused greater losses on exotic foreign exchange derivatives.

Even ignoring such potentially adverse macroeconomic effects, this episode raises many policy issues. Did the firms really understand the risks involved in the contracts? Why did they take out such large amounts of contracts? It is well known that many of these companies had bought contracts for amounts far exceeding their expected overseas revenues. Were they willingly speculating or simply fooled by the bankers structuring the deals? Should there be regulatory oversight of such complex derivatives structures? The future will tell, but in the mean time, we hope that financial managers around the world think twice before engaging in overtly complex derivative contracts.

20.6 Summary

The purpose of this chapter is to develop an understanding of futures markets and foreign exchange options markets and the use of futures and options in hedging transaction exchange risks. The main points in the chapter are as follows:

1. Foreign currency futures are standardized contracts that allow one to buy or sell specific amounts of foreign currency at a price determined today, with delivery on a given day in the future. The contracts are traded on organized exchanges.
2. The clearinghouse of an exchange is the counterparty to all transactions. To guarantee that the terms of the contracts will be met, buyers and sellers must maintain margin accounts.
3. Marking to market is the process by which the clearinghouse of an exchange debits and credits the losses and profits that arise from the daily changes in futures prices to the margin accounts.
4. Futures contracts are rarely held until delivery and are closed out by simply reversing the original transaction.
5. Futures contracts are used to hedge transaction exchange risks in a fashion similar to forward contracts. To hedge a foreign currency receivable, one must go short in that foreign currency futures contract. To hedge a foreign currency payable, one goes long in the foreign currency futures contract.
6. If the maturity of a futures contract does not coincide with the maturity of the receivable or payable to be hedged, there is basis risk.
7. Foreign currency call options give the buyer of an option the right, but not the obligation, to buy a specific amount of foreign currency at the strike price, which is an exchange rate stated in the contract. Foreign currency put options give the buyer of an option the right to sell foreign currency.
8. Foreign currency options are primarily traded in the over-the-counter interbank market, but they are also traded on exchanges.
9. Option payoffs are functions of the future spot rate. The payoff on a call option is either 0 or the difference between the spot rate and the strike price, $\max[0, S(T) - K]$; for a put option, the payoff is $\max[0, K - S(T)]$.
10. The classic use of option contracts as hedges arises in bidding situations.
11. Transaction exchange risks can be hedged with an option that gives you the right, but not the obligation, to do the transaction that gives rise to the risk.
12. Purchasing foreign currency options in hedging situations is like purchasing insurance, and varying the strike price varies the quality of the insurance.
13. Increasing the strike price of a foreign currency call (put) option decreases (increases) the option's value because it removes (adds) possible states of the world over which the contract provides revenue.
14. An increase in the variance of possible future exchange rates increases the possible range of future exchange rates for any given date in the future that increases the value of both call and put options.
15. Option prices are mostly positively related to time to maturity because an increase in time to maturity primarily increases the conditional variance of the distribution of future exchange rates.
16. Put-call parity is a no arbitrage relationship between the prices of European put and call options, the forward exchange rate, and the domestic interest rate.
17. Average-rate call options have a payoff that is the maximum of the average future exchange rate minus the strike price of the option. This is only one example of a complex payoff that can be purchased through various exotic options.

Questions

1. How does a futures contract differ from a forward contract?
2. What effects does "marking to market" have on futures contracts?
3. What are the differences between foreign currency option contracts and forward contracts for foreign currency?
4. What are you buying if you purchase a U.S. dollar European put option against the Mexican peso with a strike price of MXN10.0/\$ and a maturity of July? (Assume that it is May and the spot rate is MXN10.5/\$.)
5. What are you buying if you purchase a Swiss franc American call option against the U.S. dollar with a strike price of CHF1.30/\$ and a maturity of January? (Assume that it is November and the spot rate is CHF1.35/\$.)

6. What is the intrinsic value of a foreign currency call option? What is the intrinsic value of a foreign currency put option?
7. What does it mean for an American option to be “in the money”?
8. Why do American option values typically exceed their intrinsic values?
9. Suppose you go long in a foreign currency futures contract. Under what circumstances is your cumulative payoff equal to that of buying the currency forward?
10. What is basis risk?
11. Your CEO routinely approves changes in the fire insurance policies of your firm to protect the value of its buildings and manufacturing equipment. Nevertheless, he argues that the firm should not buy foreign currency options because, he says, “We don’t speculate in FX markets!” How could you convince him that his positions are mutually inconsistent?
12. Why do options provide insurance against foreign exchange risks in bidding situations? Why can’t you hedge with a forward contract in a bidding situation?
13. Suppose that you have a foreign currency receivable (payable). What option strategy places a floor (ceiling) on your domestic currency revenue (cost)?
14. Describe qualitatively how changing the strike price of the option provides either more or less expensive insurance.
15. Why does an increase in the strike price of an option decrease the value of a call option and increase the value of a put option?
16. Why does an increase in the volatility of foreign exchange rates increase the value of foreign currency options?
17. How does increasing time to maturity affect foreign currency option value?
18. What is the payoff on an average-rate pound call option against the dollar?
19. Suppose the current spot rate is \$1.29/€. What is your payoff if you purchase a down-and-in put option on the euro with a strike price of \$1.31/€, a barrier of \$1.25/€, and a maturity of 2 months? When would someone want to do this?

PROBLEMS

1. If you sold a Swiss franc futures contract at time *t* and the exchange rate has evolved as shown here, what would your cash flows have been?

Day	Futures Price \$/CHF	Change in Futures Price	Gain or Loss	Cumulative Gain or Loss	Margin Account
t	0.7335				
t+1	0.7391				
t+2	0.7388				
t+3	0.7352				
t+4	0.7297				

2. Given the following information, how much would you have paid on September 16 to purchase a British pound call option contract with a strike price of 155 and a maturity of October?

Data for September 16

	Calls	Puts
50,000 Australian Dollar Options (cents per unit)		
64 Oct	—	0.48
65 Oct	—	0.90
67 Oct	0.22	—
31,250 British Pounds (cents per unit)		
152.5 Dec	—	4.10
155 Oct	1.50	3.62
155 Nov	2.35	—

3. Using the data in problem 2, how much would you have paid to purchase an Australian dollar put option contract with a strike price of 65 and an October maturity?
4. Suppose that you buy a €1,000,000 call option against dollars with a strike price of \$1.2750/€. Describe this option as the right to sell a specific amount of dollars for euros at a particular exchange rate of euros per dollar. Explain why this latter option is a dollar put option against the euro.
5. Assume that today is March 7, and, as the newest hire for Goldman Sachs, you must advise a client on the costs and benefits of hedging a transaction with options. Your client (a small U.S. exporting firm) is scheduled to receive a payment of €6,250,000 on April 20, 44 days in the future. Assume that your client can borrow and lend at a 6% p.a. U.S. interest rate.
   a. Describe the nature of your client’s transaction exchange risk.
   b. Use the appropriate American option with an April maturity and a strike price of 129¢/€ to determine the dollar cost today of hedging the transaction with an option strategy. The cost of the call option is 3.93¢/€, and the cost of the put option is 1.58¢/€.

c. What is the minimum dollar revenue your client will receive in April? Remember to take account of the opportunity cost of doing the option hedge.

d. Determine the value of the spot rate ($/€) in April that would make your client indifferent ex post to having done the option transaction or a forward hedge. The forward rate for delivery on April 20 is $1.30/€.

6. Assume that today is September 12. You have been asked to help a British client who is scheduled to pay €1,500,000 on December 12, 91 days in the future. Assume that your client can borrow and lend pounds at 5% p.a.

   a. Describe the nature of your client's transaction exchange risk.

   b. What is the option cost for a December maturity and a strike price of £0.72/€ to hedge the transaction? The option premiums per 100 euros are £1.70 for calls and £2.40 for puts.

   c. What is the minimum pound cost your client will experience in December?

   d. Determine the value of the spot rate (£/€) in December that makes your client indifferent *ex post* to having done the option transaction or a forward hedge if the forward rate for delivery on December 11 is £0.70/€.

7. Assume that today is June 11. Your firm is scheduled to pay £500,000 on August 15, 65 days in the future. The current spot is $1.75/£, and the 65-day forward rate is $1.73/£. You can borrow and lend dollars at 7% p.a. Suppose you think options are overpriced because you think the dollar will be in a tight trading range in the near future. You have been thinking about selling an option as a way to reduce the dollar cost of your pound payable.

   a. If an August pound option with a strike price of 175¢/£ costs 4.5¢/£ per pound for the call and 4¢/£ for the put, what is the minimum effective exchange rate in August that you will pay? Over what range of future exchange rates will this price be achieved?

   b. How much must the pound appreciate before your speculative option strategy ends up costing you more than the forward rate?

8. Upon arriving for work on Monday, you observe a violation of put–call parity. In particular, the synthetic forward price of dollars per yen is above the current forward rate. How would you capitalize on this information?

9. Use interest rate parity to demonstrate that you can represent put–call parity as

$$P - C = \frac{K}{1 + i(\$)} - \frac{S}{1 + i(€)}$$

10. On April 28, 1995, the Paine Webber Group introduced a new type of security on the NYSE: *U.S. dollar increase warrants on the yen*. At exercise, each warrant entitled the holder to an amount of U.S. dollars calculated as

$$\text{Greater of (i) 0 and}$$
$$\text{(ii) } \$100 - [\$100 \times (¥83.65/\$/\text{Spot rate})]$$

The "spot rate" in the formula refers to the yen/dollar rate on any day during the exercise period, which extended until April 28, 1996. The 1-year forward rate on April 28 was ¥79.72/$, and the spot rate was ¥83.65/$.

   a. What view on the future yen/dollar rate do investors in this security hold?

   b. This security was issued at a price of $5.50. To see whether the security is fairly priced, which option prices would you want to examine?

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Appendix

Foreign Currency Option Pricing (Advanced)

While knowledge of option pricing models is not required to be able to use the products of financial markets, decision making is enhanced if you understand how option prices are determined. This information is especially important for quotations that involve over-the-counter market prices. Option pricing models allow us to know whether the market prices that we observe are reasonable and competitive given our perceptions of the riskiness of exchange rates. Armed with an explicit model, we also can use observable option prices to determine the implied volatility of exchange rates, which enables us to quantify the market's forecast of foreign exchange risk. This should lead to improved risk management.

While option pricing models seem quite mathematically intimidating to some, a great deal of insight into how options are priced can be gained by understanding the no-arbitrage intuition from a two-state example. The binomial model assumes that at each point in time there are only two possible states to which the exchange rate can evolve.[1] The limiting case of the binomial model as the discrete time period gets smaller and smaller is the continuous time model of Garman and Kolhagen (1983), who generalized the Black-Scholes (1973) stock-option pricing model to price foreign currency options. We present the Garman-Kolhagen model below.

A Two-State Example of Arbitrage Pricing

Suppose we want to price a 1-month European call option, which allows us to purchase £100 at a strike price of $\$1.52/£$. Let the value of this pound call option be $C(t)$ dollars today. Our goal is to determine $C(t)$.

Let the current dollar–pound spot exchange rate be $\$1.50/£$, and assume that 1 month from now, the spot rate will be either $\$1.55/£$ or $\$1.45/£$. Let's also assume that the USD interest rate is 0.5% per month and that the GBP interest rate is 1% per month.

We use an arbitrage argument to value the pound call option. We create a portfolio that perfectly replicates the possible returns on the pound call option so that the option price must equal the value of the replicating portfolio to prevent arbitrage. If the current value of the call option were greater than the value of our replicating portfolio, we could sell the call option to someone and invest in the portfolio. At maturity, we would be able to cover the outflows demanded by those who would exercise their option, and we would have wealth left over. If, on the other hand, the current value of the call option were less than the value of our replicating portfolio, we would borrow the portfolio and purchase or invest in the option. The payoff on our call option would be more than enough to offset the cost of borrowing the replicating portfolio.

To understand these arbitrage arguments, let's continue with the example. To derive our replicating portfolio we invest in £X today, and we borrow \$Y. The initial dollar cost of our replicating portfolio is therefore

$$\left[\left(\frac{\$1.50}{£}\right) \times £\text{X}\right] - \$\text{Y}$$

We must buy £X in the spot market, but we borrow \$Y, which partially offsets our dollar cost. Remember that we will get interest on our £X at 1% per month no matter what state of the world is realized in 1 month, and similarly, we will owe interest at 0.5% per month on our dollar borrowing.

If the dollar weakens, the value of the £100 call option is

$$\left[\left(\frac{\$1.55}{£}\right) - \left(\frac{\$1.52}{£}\right)\right] \times £100 = \$3.00$$

Because we have the right to buy £100 at the strike price of $\$1.52/£$ and we can sell the £100 in the spot market for $\$1.55/£$, we make \$3.00. On the other hand, if the dollar strengthens, the call option is worthless because no one wants to buy £100 at $\$1.52/£$ if the spot exchange rate is $\$1.45/£$.

From the discussion of the two payoffs on the option, we want the value of our replicating portfolio in 1 month to be

$$\left[\left(\frac{\$1.55}{£}\right) \times £\text{X} \times 1.01\right] - [\$\text{Y} \times 1.005] = \$3.00$$

[1]By extending the example to multiple periods, it generalizes to become the binomial option pricing model of Cox and Rubinstein (1985).

if the dollar weakens, and if the dollar strengthens, we want the value of the portfolio to be

$$\left[\left(\frac{\$1.45}{£}\right) \times £X \times 1.01\right] - [\$Y \times 1.005] = 0$$

The previous two equations are linear in two unknowns. Consequently, there is a unique solution for £X and \$Y.

Solving the second equation for £X gives

$$£X = \frac{\$Y \times 1.005}{\left(\frac{\$1.45}{£}\right) \times 1.01}$$

If we substitute this result into the first equation, we get

$$\left[\left(\frac{\$1.55}{£}\right) \times \left(\frac{\$Y \times 1.005}{\left(\frac{\$1.45}{£}\right) \times 1.01}\right) \times 1.01\right] - [\$Y \times 1.005] = \$3.00$$

Solving this equation for \$Y gives

$$\$Y = \$43.28$$

Substituting into the solution for £X gives

$$£X = £29.70$$

Hence, the cost of the replicating portfolio is

$$\left[\left(\frac{\$1.50}{£}\right) \times £29.70\right] - \$43.28 = \$1.27$$

Consequently, because this portfolio replicates the payoff on the £100 call option, the dollar cost of this option must be \$1.27 to prevent arbitrage.

Suppose that the £100 call option were more expensive than \$1.27, say \$1.37. In this situation, selling the call option and investing the proceeds in the replicating portfolio should make money. From earlier, we know that if we borrow \$43.28 and lend £29.70, we will replicate the payoffs on the £100 call option because

$$\left[\left(\frac{\$1.55}{£}\right) \times £29.70 \times 1.01\right] - (\$43.28 \times 1.005) = \$3.00$$

and

$$\left[\left(\frac{\$1.45}{£}\right) \times £29.70 \times 1.01\right] - (\$43.28 \times 1.005) = 0$$

Consequently, we will be able to meet the demands of the investors who purchased the call option from us. But, the cost of the replicating portfolio is only \$1.27, whereas we generate \$1.37 by selling the £100 call option. Clearly, given these prices, we would try to sell as many of these call options as possible, investing the proceeds in the replicating portfolio to cover the demands of our investors, but keeping the residual for ourselves.

Conversely, if the price of the £100 call option were less than \$1.27, we would make money by doing exactly the opposite set of transactions. We would buy the call options and lend the replicating portfolio. Suppose the price of the £100 call option were \$1.20. If we borrow £29.70 at 1% per month, and convert the pounds into dollars, we get

$$\left(\frac{\$1.50}{£}\right) \times £29.70 = \$44.55$$

We can buy the £100 pound call option for \$1.20, which leaves

$$\$44.55 - \$1.20 = \$43.35$$

to invest at 0.5% per month. In 1 month, we will have to pay back interest and principal on our pound borrowing; we can collect interest and principal on the dollars we invested; and we can collect the payoff on our pound call options. Hence, we will have either

$$-\left[\left(\frac{\$1.55}{£}\right) \times £29.70 \times 1.01\right] + (\$43.35 \times 1.005) + \left[\left(\frac{\$1.55}{£}\right) - \left(\frac{\$1.52}{£}\right)\right] \times £100 = \$0.07$$

or

$$-\left[\left(\frac{\$1.45}{£}\right) \times £29.70 \times 1.01\right] + (\$43.35 \times 1.005) = \$0.07$$

Because we have generated \$0.07 in both states of the world without any investment of our own money, we have a riskless arbitrage, and we would try to invest on a much larger scale.

Notice in this example that we did not need to know the probabilities associated with the possible up and down movements in the exchange rate. We only needed to know the current spot rate, the strike price, the two interest rates, and the two possible values of the future spot rate. The fact that the probabilities were irrelevant means that the expected rate of appreciation of the pound relative to the dollar was not directly relevant to determining the value of the pound call option.

You may find this to be a counterintuitive result, especially when reflecting on the earlier discussion in this chapter, which indicated that the value of a call option depends on the position of the strike price in relation to the probability distribution of the future exchange rate.

The puzzle arises because one naturally thinks that the probability distribution of future exchange rates depends on the mean rate of appreciation of one currency relative to another. While the intuition in this chapter is correct, the explanation for why the option does not depend explicitly on the mean rate of appreciation in this example is that we were able to price the option by a no-arbitrage argument. Essentially, the spot exchange rate, the interest rates, and the volatility of the process driving exchange rates implicitly characterize the future distribution. Formally, option pricing is said to rely on risk-neutral pricing.[2] Whenever we are able to develop a no-arbitrage argument, option prices will not depend explicitly on the expected rate of appreciation, and we are able to price options with risk-neutral methods.

The Binomial Option Pricing Model

The last section considered an example in which there were only two possible values for the future exchange rate. While this is clearly unrealistic over an interval as long as a month, it may not be so unreasonable over a very short time interval. **Binomial option pricing** relies on the assumption that random movements in the underlying asset, in this case the exchange rate, over short intervals are well approximated by a discrete, two-state model.[3]

To develop the intuition, let the spot exchange rate, $S(t)$, denote the domestic currency price of the foreign currency. At each discrete point in time t, it is assumed that $S(t)$ will either move up to $uS(t)$ or down to $dS(t)$ at time $t+1$. Analogously, we can assume that the domestic currency price of a call option to purchase one unit of the foreign currency, $C(t)$, will evolve up to $C(u, t+1)$ if the exchange rate increases, or down to $C(d, t+1)$ if the exchange rate decreases. If there were only one period left before the maturity of the option, we would use the logic of the numerical example to value the call option.

We could form a portfolio containing an investment of $\Delta(t)$ units of foreign currency in the foreign currency–denominated risk-free asset and $B(t)$ units of domestic currency in the domestic currency–denominated risk-free asset. Earlier, we found $B(t) < 0$, in which case we are borrowing the domestic currency. The domestic currency cost of this replicating portfolio would be

$$S(t)\Delta(t) + B(t)$$

As above, if the call price, $C(t)$, were not equal to the value of the replicating portfolio, there would be an arbitrage opportunity.

It is straightforward, although tedious, to add more periods. The simplest binomial model assumes that the possible events in each period are independent of all previous events and that the sizes of the possible increases or decreases in the exchange rate are the same in all future periods. Hence, if there are two periods remaining, there are three possible values for the exchange rate at the maturity of the option. The exchange rate can increase twice, it can decrease twice, or it can first increase and then decrease, which is the same as first decreasing and then increasing. Similarly, the price of the call option will either increase twice, decrease twice, first increase and then decrease, or first decrease and then increase. This binomial tree is illustrated in Exhibit 20A.1.

The value of the call option in the first period, $C(t)$, can then be determined by an argument that uses the technique of backward recursion. We know that in the next to the last period, we will have evolved to a particular state, and there will be only two possible events that characterize the last period. Therefore, we can use the logic of the replicating portfolio that was developed earlier to determine the possible call prices for the next to the last period. From there, we work backward to develop the possible prices in previous periods.

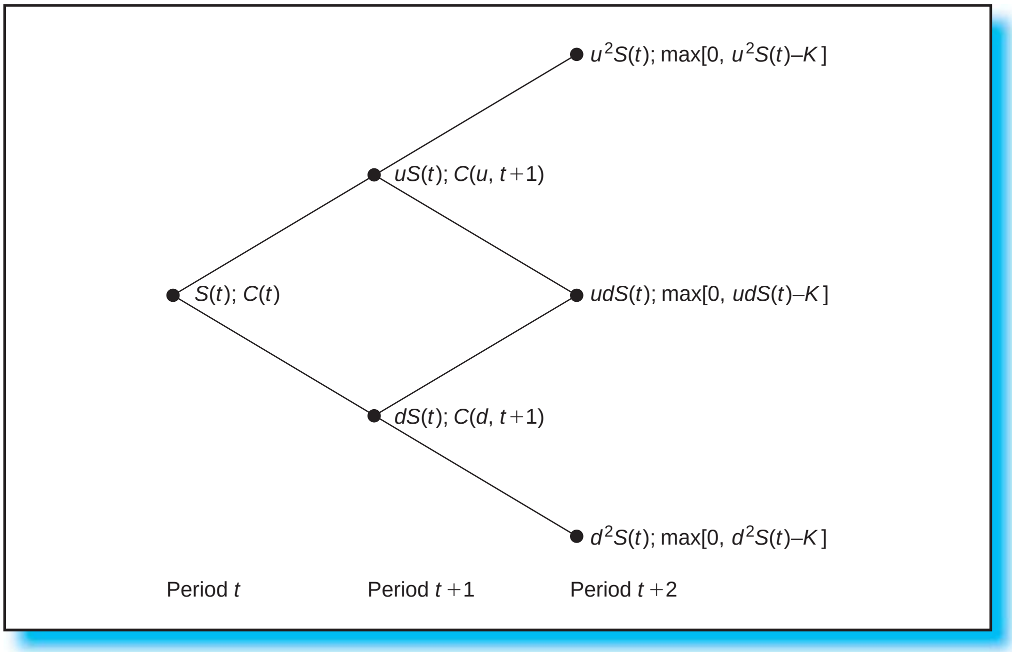
The Continuous Time Case

Let T be the calendar time to expiration of the option contract, and let the continuously compounded domestic and foreign interest rates for that maturity be $i(t)$ and $i^*(t)$, respectively. Let the volatility of the rate of depreciation over a small time interval h be $\sigma\sqrt{h}$. That is, σ is the annualized standard deviation, and h is the fraction of the year. In the earlier discrete time analysis, we thought of dividing the interval of time T into n periods, each of length h, where $h = T/n$. If we drive n to infinity, the period h shrinks to 0, and the exchange rate is said to follow a continuous time stochastic process. The resulting expression for the call option price is the Garman-Kolhagen version of the Black-Scholes option pricing model.

While some of you may be interested in the formal arguments that lead to the option pricing formula, we

[2]Remember, a risk-neutral investor is indifferent between holding an asset with a certain return and holding a different asset that has an uncertain return if its expected return is the same as the risk-free return.
[3]See Cox and Rubinstein (1985) for additional discussion of the binomial option pricing model.

Exhibit 20A.1 A Two-Period Binomial Tree



will not present those here. Instead, we present the final formula and discuss some intuition. The price of a call option in the Garman-Kolhagen model is

$$C(t) = \exp(-i^*(t)T)\, S(t)\, N(d_1(t)) \\ -\exp(-i(t)T)K\, N(d_2(t))$$

The terms $N(d_1(t))$ and $N(d_2(t))$ are probabilities associated with the cumulative standardized normal distribution. That is

$$N(d) = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$$

The limits of integration are

$$d_1(t) = \left[\ln[S(t)/K] + [i(t) - i^*(t) \\ + 0.5\sigma^2]T\right]/\sigma\sqrt{T}$$

and

$$d_2(t) = d_1(t) - \sigma\sqrt{T}$$

Exhibit 20A.2 presents various prices for European call options corresponding to plausible values of the variables entering the Garman-Kolhagen model. The option prices and the exchange rates are expressed in U.S. cents per pound, as on the NASDAQ OMX PHLX. Hence, the spot exchange rate is measured as 150¢/£. The strike prices represent in-the-money call options with a strike price of 145¢/£, at-the-money calls with a strike price of 150¢/£, and out-of-the-money calls with a strike price of 155¢/£. The volatilities range from 8% per annum, which is relatively low, to a more normal value of 12%, and a relatively high value of 16%. Finally, the U.S. dollar interest rate takes the values of 1%, 4%, and 7% per annum. The pound interest rate is held constant at 5% per annum.

The option prices are larger the lower is the strike price, the higher is the volatility, the higher is the U.S. interest rate, and, usually, the longer is the time to maturity. Note that for the in-the-money call options with a low U.S. interest rate and low volatility, the call option prices actually decrease with longer times to expiration. These facts will be explored in more detail later.

Notice that the replicating portfolio for the binomial option is $C(t) = S(t)\Delta(t) + B(t)$. In the continuous time case, the same expression equates the call option price to the value of the portfolio that replicates the instantaneous payoff on a call option. Hence, by comparing the option pricing formula to the price of the call option, we see that the domestic currency investment in the foreign currency risk-free asset is $\exp(-i^*(t)T)\, S(t)\, N(d_1(t))$, and the amount of domestic currency that is borrowed is $\exp(-i(t)T)\, K\, N(d_2(t))$. These amounts change continuously as the values of the parameters underlying the option prices change.

Once we have call option prices, we can use put–call parity to determine the value of put options.

Exhibit 20A.2 Garman-Kolhagen Call Option Values

Spot = 150¢/£, $i^{£}$ = 5%

Vol.	Strike	$i^{\$}$ = 1%			$i^{\$}$ = 4%			$i^{\$}$ = 7%		
		Fraction of a Year to Maturity								
σ	**K**	**0.25**	**0.50**	**0.75**	**0.25**	**0.50**	**0.75**	**0.25**	**0.50**	**0.75**
8%	145	4.49	4.39	4.27	5.30	5.75	6.10	6.16	7.29	8.27
8%	150	1.70	2.06	2.23	2.18	2.95	3.48	2.74	4.06	5.12
8%	155	0.44	0.80	1.02	0.54	1.27	1.77	0.86	1.91	2.86
12%	145	5.53	5.99	6.26	6.24	7.24	7.98	7.00	8.61	9.92
12%	150	2.87	3.66	4.13	3.36	4.60	5.48	3.91	5.68	7.07
12%	155	1.27	2.07	2.59	1.55	2.72	3.59	1.88	3.49	4.82
16%	145	6.62	7.61	8.24	7.29	8.79	9.89	7.99	10.07	11.70
16%	150	4.04	5.29	6.09	4.54	6.25	7.47	5.09	7.31	9.03
16%	155	2.26	3.53	4.38	2.61	4.27	5.51	2.99	5.11	6.81

Note: The maturities of the options are 91 days, 182 days, or 273 days with a 365-day year.

Comparative Statics for the Call Option Price

We next consider how call option prices are affected by changes in the various variables that determine the price. The effects of changes in the underlying variables on the option prices are partial derivatives since they are derived holding the influence of the other variables constant. Often, financial market participants discuss these effects with Greek letters.

To help in understanding the nature of each partial effect, we will discuss how the price changes relative to a base case whose parameter values are the following:

Spot exchange rate = 150¢/£
Strike price = 152¢/£
USD interest rate = 3% per annum
GBP interest rate = 4% per annum
Volatility = 12% per annum
Time to maturity of 0.25 years

These are reasonable values that one might encounter in actual markets, and the theoretical value of a call option with these parameter values is 2.5224¢/£.

The Delta of an Option

The first partial effect examines how the call option price changes when the exchange rate changes. From the derivation of the call option price, we find

$$\frac{\partial C(t)}{\partial S(t)} = \Delta(t) = \exp(-i^{*}(t)T)\, N(d_1(t)) > 0$$

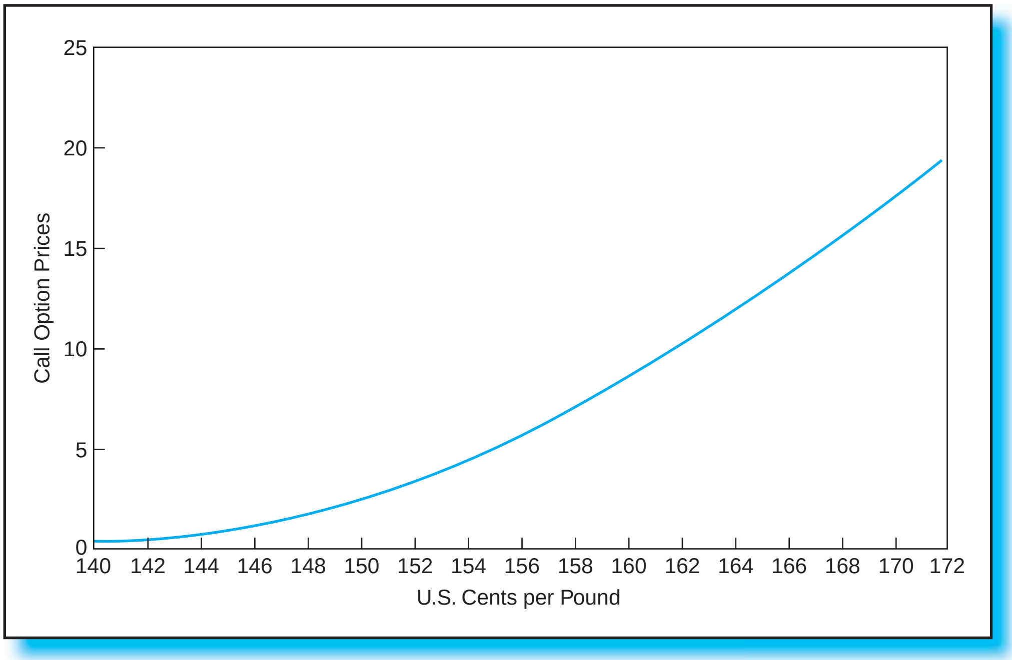
This expression is called the **delta** of the call option because it represents the change in the value of the derivative asset with a small change in the value of the underlying asset. The expression is also sometimes called the "hedge ratio" because it arises from the construction of the replicating portfolio. Delta is the amount of pounds invested in the risk-free asset to replicate the payoff on the call option. Evaluating this equation at the base parameter values gives a delta of 0.4039. Hence, if the exchange rate increases from 150.00¢/£ to 150.01¢/£, the value of the call option increases by (0.4039)(0.01¢/£) = 0.0040¢/£, or from 2.5224¢/£ to 2.5264¢/£.

Exhibit 20.14 graphs the call option value as a function of the exchange rate. The slope of the curve in Exhibit 20A.3 is the delta of the call option. Notice that when the exchange rate is low relative to the strike price of 152¢/£, the call option is deeply out of the money, and the delta of the option is nearly 0 because increases in the exchange rate have only a small effect on the option value. As the exchange rate increases toward the strike price, the sensitivity of the option value to changes in the exchange rate increases. Eventually, when the exchange rate is well above the exercise price, the delta of the call option approaches 1 as the option value nearly increases one-for-one with an increase in the exchange rate. To summarize, we have found that the delta of a call option is always between 0 and 1 because the change in the option price is less than the change in the underlying spot rate.

Delta Hedging

Knowledge of the delta of an option is important for those who sell or write options because it tells them how

Exhibit 20A.3 Call Option Price



to hedge the position that they have created. Suppose we have sold call options on one million pounds, and we do not want to be exposed to losses from movements in the underlying exchange rate. Then, if the pound call option against the dollar has a delta of 0.34, we know that we must own $0.34 \times$ £1,000,000 = £340,000 to have a hedged position. If alternatively, we have sold £1,000,000 put options whose delta is −0.48, we will have to borrow $0.48 \times$ £1,000,000 = £480,000 to have a hedged position.

Making a market in call and put options without losing a lot of money requires the trader to be continually aware of the exposure to losses due to fluctuations in exchange rates that is inherent in the trader's portfolio at any point in time. Essentially, the trader must track the delta of his overall portfolio created through his transactions. This is not too difficult because deltas are additive. Hence, the overall exposure of the portfolio is the sum of the different deltas of each of the options that is bought or sold weighted by the amount of the position.

For example, if a trader sells a call option on £1,000,000 with a delta of 0.40 and buys another call option on £1,000,000 with different parameters whose delta is 0.45, his net exposure to small movements in the exchange rate is a delta of $0.05 = 0.45 - 0.40$. Effectively, selling the one call option coupled with buying the other call option leaves the trader with a net long position of $0.05 \times$ £1,000,000 = £50,000. The trader's portfolio of options will lose value if the pound weakens relative to the dollar.

Many market makers attempt to remain reasonably close to **delta neutral**, which means that they actively adjust their portfolios so that they are not exposed to risk of loss from small changes in foreign exchange rates. In the previous example, in which the trader is effectively long £50,000, one way to achieve a delta-neutral position would be to sell £100,000 of call options with a delta of 0.5. Obviously, there are many ways in which a delta-neutral position could be established, and profitable market making involves trying to buy options that are undervalued and to sell options that are overvalued while managing the exposure to exchange rates and the other variables that affect option prices.

The Gamma of an Option

The **gamma of a call option** describes how the option's delta changes with a change in the underlying exchange rate:

$$\gamma(t) = \frac{\partial \Delta(t)}{\partial S(t)} = \frac{\exp(-i^*(t)T)\phi(d_1(t))}{S(t)\sigma\sqrt{T}}$$

where $\phi(z)$ represents the probability density function of the standard normal:

$$\phi(z) = \frac{1}{\sqrt{2\pi}}\exp\left(\frac{-z^2}{2}\right)$$

When the option is either deeply out of the money or deeply in the money, the gamma of the call option is near 0. When the exchange rate is near the strike price, gamma takes its largest values. When evaluated at the base parameter values given above, the delta of the option is 0.4039, and the value of gamma is 0.04. Hence, if the exchange rate changes from 150.00¢/£ to 150.01¢/£, the delta of the call option increases by $0.04 \times 0.01 = 0.0004$, or from 0.4039 to 0.4043.

Knowledge of the gamma of an option is important to those who sell or write options because it tells them how rapidly they must adjust their hedge ratios when the exchange rate changes. For example, if a call option that has been sold is deep in the money, the hedge ratio is near 1, and the gamma is very low. Holding the underlying asset in an amount equivalent to the amount of the call option is effectively what is required to produce a riskless position. Even if the exchange rate changes, no change in the underlying position is required.

But, if the call option is at the money, the hedge ratio is much less than 1, and the gamma is large. An increase in the exchange rate may move the delta of the option from 0.40 to 0.50. Now, if the trader had sold £1,000,000 of call options and was holding £400,000 as a hedge, the trader must buy an additional £100,000 if he does not want to bear exchange risk. An understanding of gamma helps the trader determine in advance how much of which options will have to be bought or sold after the next movement in the exchange rate.

The Elasticity of an Option

The elasticity of a call option price with respect to the exchange rate is denoted in various presentations as either omega, $\Omega(t)$, or lambda, $\lambda(t)$. Elasticity measures the percentage change in the call option price divided by the percentage change in the exchange rate:

$$\Omega(t) = \frac{\dfrac{\partial C(t)}{C(t)}}{\dfrac{\partial S(t)}{S(t)}} = \Delta(t)\frac{S(t)}{C(t)}$$

For example, the delta of the call option at the basic parameters is 0.4039, the current spot rate is 150¢/£, and the price of the call option is 2.52¢/£. Thus, the elasticity of this call option is $\Omega(t) = .4039 \times \dfrac{150}{2.52} = 24.04$. Hence, if the exchange rate increases by 1% to 151.5¢/£, the price of the call option is predicted to increase by 24.04% to $3.13¢/£ = (2.52¢/£) \times 1.2404$.

The elasticity of a call option is an important concept because the volatility of a call option is its elasticity times the volatility of the exchange rate. Because the volatility of the underlying rate of appreciation is 12%, the volatility of the call option is $24.04 \times 12\% = 288.48\%$. Unhedged positions in foreign currency call options are quite volatile.

The Vega of an Option

The change in the value of a call option as volatility changes is usually referred to as the vega of the option. Increasing the volatility of exchange rates increases the value of the call option because

$$\frac{\partial C(t)}{\partial \sigma} = \exp(-i^*(t)T)\phi(d_1(t))\sqrt{T} > 0$$

The value of the call option is quite sensitive to the volatility of the exchange rate. For example, the vega of the option at the base parameters is 29. Hence, a change in volatility from 12% to 13% increases the option value by $0.01 \times 29 = 0.29$ or from 2.52¢/£ to 2.81¢/£.

The Rhos of an Option

The changes in the value of a call option as either the domestic or foreign interest rates change are usually referred to as the rhos of the option. The price of a foreign currency call option increases with the domestic interest rate and decreases with the foreign interest rate because

$$\frac{\partial C(t)}{\partial i(t)} = T\exp(-i(t)T)\,K\,N(d_2(t)) > 0$$

and

$$\frac{\partial C(t)}{\partial i^*(t)} = -T\exp(-i^*(t)T)\,S(t)\,N(d_1(t)) < 0$$

When these equations are evaluated at the base parameter values, their values are 0.14 and −0.15, respectively, when the interest rates are expressed as percent per annum. Thus, for example, an increase in the USD interest rate from 3% to 4% increases the option price from 2.52¢/£ to 2.66¢/£ or by 0.14¢/£. Similarly, an increase in the GBP interest rate from 4% to 5% decreases the option price from 2.52¢/£ to 2.37¢/£ or by 0.15¢/£.

The Theta of an Option

The last effect that we discuss is how changes in the maturity of an option affect its value. As noted earlier, increasing the time to maturity has an ambiguous effect on the call option price. Formally, we have

$$\begin{aligned}\frac{\partial C(t)}{\partial T} &= -i^*(t)\exp(-i^*(t)T)S(t)\,N(d_1(t))\\ &\quad + i(t)\exp(-i(t)T)\,K\,N(d_2(t))\\ &\quad + \exp(-i^*(t)T)\,S(t)\phi(d_1(t))\sigma/(2\sqrt{T})\end{aligned}$$

The expression may be negative for call options that are in the money ($S_t > K$) especially if they are deep in the money or the time to maturity is short. The situation is exacerbated if the foreign interest rate is well above the domestic interest rate. If the foreign interest rate were zero, an increase in time to maturity would increase the foreign currency option value.

Rather than discuss the sensitivity of call option prices to an increase in maturity, market participants are interested in the sensitivity of the price of a call option to the passage of time, which is often referred to as the option's **theta**. The theta of a call option is simply the negative of the derivative of the call option with respect to maturity, and it describes how the option price will evolve as the time remaining until maturity decays.

To examine the influence of time to maturity on call option value at the base case parameter values, we calculate the theta of the call option as 1 day elapses. The call option with 91 days to maturity is worth 2.5224¢/£, while the call option with 90 days to maturity is worth 2.5052¢/£. This corresponds with the theta of the 91-day option being −0.0171¢/£.

Implied Volatility

Since all of the variables that determine foreign currency option prices are observable except the volatility of the exchange rate, option prices can be used in conjunction with an option pricing model and the observations on the other variables to determine an **implied volatility**. This is the unique value of σ that sets the option price from the model equal to the option price observed in the market.

There are several ways that one may determine an implied volatility, but the simplest is merely to try out a value, say $\sigma = 0.11$, and see if the price from the option pricing model is higher or lower than the observed option price from the market. If the model's price is too high, we know that we must lower the implied volatility. If the model's price is too low, we must increase the implied volatility. It isn't too hard to iterate and find the unique solution.[4]

One important use of implied volatility is to determine if one option is expensive relative to another option with the same maturity but with a different strike price. Since the options are for the same maturity, they are pricing the same distribution of future spot rates, and they should have the same implied volatility. Of course, one reason that the implied volatilities may differ is that one or more of the assumptions of the option pricing model may be wrong.

Summary of the Appendix

1. The theory associated with foreign currency call option pricing uses a replicating portfolio that consists of an investment in the foreign currency risk-free asset that is partially financed by borrowing the domestic currency. The payoffs on the replicating portfolio are constructed to match the payoffs on a foreign currency call option, and the value of the replicating portfolio equals the value of the option to prevent arbitrage.
2. If it is assumed that the continuously compounded rate of appreciation of the foreign currency relative to the domestic currency is normally distributed, the binomial model converges to the Garman-Kolhagen model in the limit as the number of periods between the current date and the expiration of the option goes to infinity.
3. The delta of a call option represents the change in the value of the option with a small change in the value of the underlying spot exchange rate. The expression is also sometimes called the "hedge ratio" because it is equal to the amount of foreign currency in the replicating portfolio. Delta must be between 0 and 1.
4. If a portfolio of foreign currency options does not change in value with a change in the exchange rate, the portfolio is said to be delta neutral.
5. The implied volatility of an option is the unique value of the standard deviation of the rate of appreciation that sets the option price derived from a model equal to the observed market price given observations from market prices on the other variables that affect the option pricing formula.

[4] Of course, most modern spreadsheet programs contain an equation solver that will do the iterations for you. Hence, it is a simple matter to let the computer determine the unique value of implied volatility.

Additional Questions

1. Explain intuitively how foreign currency options can be replicated with portfolios of borrowing and lending in the two currencies.
2. Why do the formulas for option prices not depend explicitly on the expected rate of appreciation of one currency relative to another currency?
3. What is the Garman-Kolhagen model of foreign currency option pricing?
4. What is the delta of an option? Why is it useful?
5. What does it mean for a portfolio of options to be delta neutral?
6. What is the gamma of an option?
7. How does a change in the volatility of the rate of appreciation affect the pricing of foreign currency options?
8. Why does a change in the domestic interest rate affect the pricing of a foreign currency option? Does a change in the foreign interest rate cause any change in option prices?
9. What is the theta of an option?
10. What is the implied volatility of an option?

Additional Problems

1. Let the current spot rate be \$1.25/€, and assume that 1 month from now the spot rate will be either \$1.30/€ or \$1.20/€. Let the dollar interest rate be 0.4% per month, and let the euro interest rate be 0.3% per month. Develop a portfolio that replicates the payoff on a 1-month euro call option with a strike price of \$1.25/€. What is the corresponding price of the euro put option with the same strike price?
2. Suppose that the price of the euro call option in Problem 1 were \$0.03/€. How would you arbitrage between the market in risk-free assets and the foreign currency options market? What would you do if the price of the call option were \$0.02/€?
3. Let the continuously compounded 6-month USD interest rate be 3% p.a., let the analogous JPY interest rate be 1% p.a., let the exchange rate be ¥98/\$, and assume that the volatility of the continuously compounded annualized rate of appreciation of the yen relative to the dollar is 13%. Use the Garman-Kolhagen option pricing model to determine the yen price of a 6-month European dollar call option with a strike price of ¥100/\$. How does your answer change if the volatility were 16% p.a.?
4. With the same variables as in Problem 3, use put–call parity to determine the yen price of the corresponding dollar put option with the same maturity and same strike price.
5. Suppose a trader sells a call option on £500,000 with a delta of 0.35 and buys another call option on £1,000,000 with different parameters whose delta is 0.55. What is his net exposure to small movements in the exchange rate? How could he cover this position?
6. Assume you are looking at prices from the NASDAQ OMX PHLX and that the price of a 3-month European AUD call option with a strike price of 92 cents per Australian dollar is 3.2¢/AUD. Suppose that the spot exchange rate is 90¢/AUD, the continuously compounded annualized dollar interest rate is 2%, and the analogous AUD interest rate is 5%. What is the implied volatility of the continuously compounded annualized rate of appreciation of the AUD relative to the dollar?
7. Suppose the implied volatilities expressed in percent per annum of yen call options against the dollar with maturities of 1, 2, and 3 months are 9%, 10%, and 11%, respectively. If you thought that the market would soon price options to have a common volatility of 10%, what position would you take in the options to expect to profit from your beliefs?

Additional Bibliography

Cox, John C., and Mark Rubenstein, 1985, *Option Markets*, Upper Saddle River, NJ: Prentice Hall.